Σελίδα 83 - Έλλειψη

3 (α) $\frac{x^2}{12}+\frac{y^2}{4}=1 \Rightarrow \frac{2x}{12_6}+\frac{2y}{4_2}y'=0 \Rightarrow \frac{y}{2}y'=-\frac{x}{6}$.

$\Rightarrow y'=-\frac{2x}{6y}=-\frac{x}{3y} \Rightarrow \lambda_{εφ}=-\frac{3}{3.1}=-1 \Rightarrow \lambda_κ=1$

$\Rightarrow y-1=-1(x-3) \Rightarrow y-1=-x+3 \Rightarrow \boxed{y=-x+4}$ εξ. εφαπτομ.

$y-1=1(x-3) \Rightarrow y-1=x-3 \Rightarrow \boxed{y=x-2}$ εξ. καθετης.

(β) $4x^2+9y^2=72 \Rightarrow 8x+18yy'=0 \Rightarrow y'=-\frac{8x}{18y}=-\frac{4x}{9y}$

$\Rightarrow \lambda_{εφ}=-\frac{\overset{2}{\cancel{4}}.\cancel{3}}{\underset{3}{\cancel{9}}.\cancel{x}}=-\frac{2}{3}// \Rightarrow \lambda_κ=\frac{3}{2}$

$\Rightarrow y-2=-\frac{2}{3}(x-3) \Rightarrow 3y-6=-2x+6 \Rightarrow \boxed{3y=-2x+12}$
εξ. εφαπτομένης

$y-2=\frac{3}{2}(x-3) \Rightarrow 2y-4=3x-9 \Rightarrow \boxed{2y=3x-5}$
εξ καθετης.

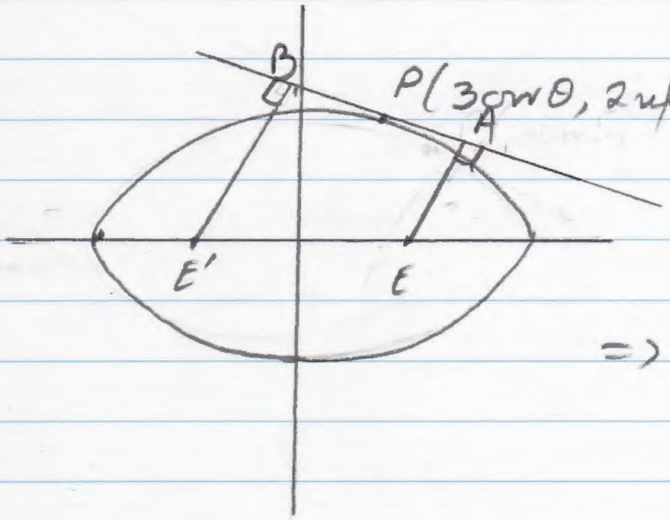
(γ) $2x^2+y^2=4 \Rightarrow 4x+2yy'=0 \Rightarrow y'=-\frac{4x}{2y}=-\frac{2x}{y}$

$(1,κ) \Rightarrow 2+κ^2=4 \Rightarrow κ^2=2 \Rightarrow κ=\pm\sqrt{2} \Rightarrow \boxed{κ=\sqrt{2}}$

$\Rightarrow \lambda_{εφ}=-\frac{2.1}{\sqrt{2}}=-\sqrt{2} \Rightarrow y-\sqrt{2}=-\sqrt{2}(x-1) \Rightarrow y-\sqrt{2}=-\sqrt{2}x+\sqrt{2}$
$\Rightarrow \boxed{y=-\sqrt{2}x+2\sqrt{2}}$ εξ εφαπ

$\lambda_κ=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2} \Rightarrow y-\sqrt{2}=\frac{\sqrt{2}}{2}(x-1) \Rightarrow 2y-2\sqrt{2}=\sqrt{2}x-\sqrt{2} \Rightarrow \boxed{2y=\sqrt{2}x+\sqrt{2}}$
εξ. καθετης.

(4) $4x^2+9y^2=36 \quad \Rightarrow \frac{x^2}{9}+\frac{y^2}{4}=1$



$P(3\sigma\upsilon\nu\theta, 2\eta\mu\theta)$

$\Rightarrow a=3$
$b=2$
$\gamma^2=9-4=5 \Rightarrow \boxed{\gamma=\sqrt{5}}$

$\Rightarrow E(\sqrt{5},0)$ και $E'(-\sqrt{5},0)$

$$8x+18yy'=0 \Rightarrow y'=-\frac{8x}{18y} \Rightarrow y'=-\frac{4x}{9y}$$

$$\Rightarrow \lambda_{\varepsilon\varphi_P} = -\frac{\cancel{4}^{2}\cdot\cancel{3}\sigma\upsilon\nu\theta}{_{3}\cancel{9}\cdot\cancel{2}\eta\mu\theta} = -\frac{2\sigma\upsilon\nu\theta}{3\eta\mu\theta} \;//$$

$\Rightarrow$ Εξίσωση εφαπτομένης στο P: $y-2\eta\mu\theta=-\frac{2\sigma\upsilon\nu\theta}{3\eta\mu\theta}(x-3\sigma\upsilon\nu\theta)$

$$\Rightarrow 3\eta\mu\theta\cdot y-6\eta\mu^2\theta=-2\sigma\upsilon\nu\theta\cdot x+6\sigma\upsilon\nu^2\theta$$

$$\Rightarrow \boxed{3\eta\mu\theta\cdot y+2\sigma\upsilon\nu\theta\cdot x=6}$$

$$\Rightarrow E'B=\frac{|3\eta\mu\theta\cdot 0+2\sigma\upsilon\nu\theta(-\sqrt{5})-6|}{\sqrt{9\eta\mu^2\theta+4\sigma\upsilon\nu^2\theta}} = \frac{|-2\sqrt{5}\sigma\upsilon\nu\theta-6|}{\sqrt{9\eta\mu^2\theta+4\sigma\upsilon\nu^2\theta}}$$

$$EA=\frac{|3\eta\mu\theta\cdot 0+2\sigma\upsilon\nu\theta\cdot\sqrt{5}-6|}{\sqrt{9\eta\mu^2\theta+4\sigma\upsilon\nu^2\theta}} = \frac{|2\sqrt{5}\sigma\upsilon\nu\theta-6|}{\sqrt{9\eta\mu^2\theta+4\sigma\upsilon\nu^2\theta}}$$

$$\Rightarrow (EA)\cdot(E'B)=\frac{|(-6)^2-(2\sqrt{5}\sigma\upsilon\nu\theta)^2|}{9\eta\mu^2\theta+4\sigma\upsilon\nu^2\theta} = \frac{|36-20\sigma\upsilon\nu^2\theta|}{9(1-\sigma\upsilon\nu^2\theta)+4\sigma\upsilon\nu^2\theta}$$

$$=\frac{36-20\sigma\upsilon\nu^2\theta}{9-5\sigma\upsilon\nu^2\theta}=\frac{4\cancel{(9-5\sigma\upsilon\nu^2\theta)}}{\cancel{9-5\sigma\upsilon\nu^2\theta}}=4 \;//$$

$\Rightarrow$ ανεξάρτητο του θ.

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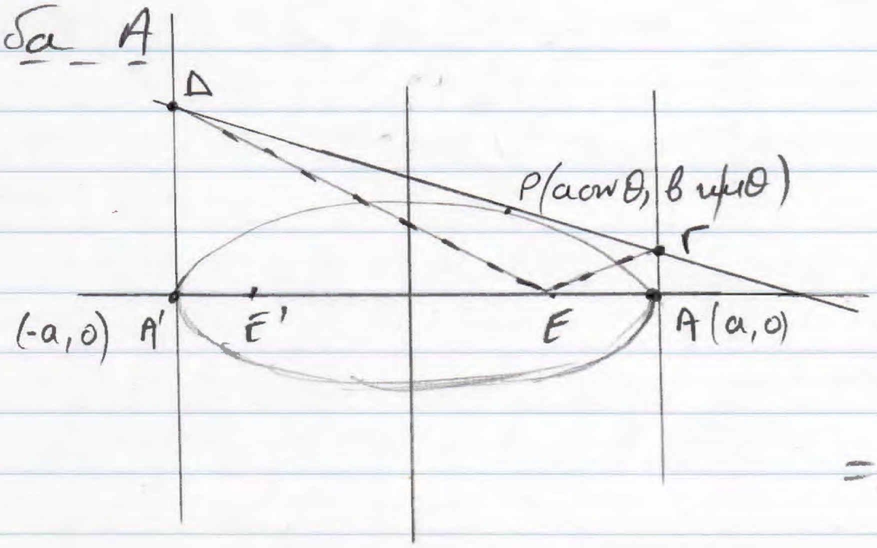
Ασκήσεις έλλειψης σελ 84

Ομάδα Α

(5)



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{2x}{a^2}+\frac{2y}{b^2}y'=0$$

$$\Rightarrow y'=-\frac{\frac{2x}{a^2}}{\frac{2y}{b^2}}=-\frac{xb^2}{ya^2}$$

$\boxed{\Delta(-a, y_\Delta)}$ $\boxed{\Gamma(a, y_\Gamma)}$

$$\Rightarrow \lambda_{\varepsilon_P}=-\frac{a\sigma\upsilon\nu\theta\, b^2}{b\eta\mu\theta\cdot a^2}=-\frac{b\sigma\upsilon\nu\theta}{a\eta\mu\theta}.$$

$\Rightarrow$ Εξίσωση εφαπτομένης στο Ρ: $y-b\eta\mu\theta=-\dfrac{b\sigma\upsilon\nu\theta}{a\eta\mu\theta}(x-a\sigma\upsilon\nu\theta)$

$$\Rightarrow ya\eta\mu\theta-ab\eta\mu^2\theta=-b\sigma\upsilon\nu\theta\cdot x+ab\sigma\upsilon\nu^2\theta$$

$$\Rightarrow \boxed{ya\eta\mu\theta+b\sigma\upsilon\nu\theta\cdot x=ab}$$

$\Rightarrow x_\Delta=-a \Rightarrow ya\eta\mu\theta+b\sigma\upsilon\nu\theta\cdot(-a)=ab \Rightarrow$

$\Rightarrow y\eta\mu\theta=b+b\sigma\upsilon\nu\theta \Rightarrow \boxed{y_\Delta=\dfrac{b(1+\sigma\upsilon\nu\theta)}{\eta\mu\theta}}$

$\Rightarrow x_\Gamma=a \Rightarrow ya\eta\mu\theta+b\sigma\upsilon\nu\theta\cdot a=ab$

$\Rightarrow y\eta\mu\theta=b-b\sigma\upsilon\nu\theta \Rightarrow \boxed{y_\Gamma=\dfrac{b(1-\sigma\upsilon\nu\theta)}{\eta\mu\theta}}$

$$\left.\begin{aligned}(A\Gamma)&=|y_\Gamma|=\frac{b(1-\sigma\upsilon\nu\theta)}{\eta\mu\theta}\\(A'\Delta)&=|y_\Delta|=\frac{b(1+\sigma\upsilon\nu\theta)}{\eta\mu\theta}\end{aligned}\right\}\Rightarrow (A\Gamma)(A'\Delta)=\frac{b^2(1-\sigma\upsilon\nu^2\theta)}{\eta\mu^2\theta}=\frac{b^2\eta\mu^2\theta}{\eta\mu^2\theta}=b^2$$

(β) $$\lambda_{E\Gamma} = \frac{y_\Gamma - y_E}{x_\Gamma - x_E} = \frac{\frac{b(1-\sigma\upsilon\nu\theta)}{\eta\mu\theta} - 0}{a - \gamma} = \frac{b(1-\sigma\upsilon\nu\theta)}{\eta\mu\theta(a-\gamma)}$$

$$\lambda_{E\Delta} = \frac{y_\Delta - y_E}{x_\Delta - x_E} = \frac{\frac{b(1+\sigma\upsilon\nu\theta)}{\eta\mu\theta} - 0}{-a - \gamma} = \frac{b(1+\sigma\upsilon\nu\theta)}{\eta\mu\theta(-a-\gamma)}$$

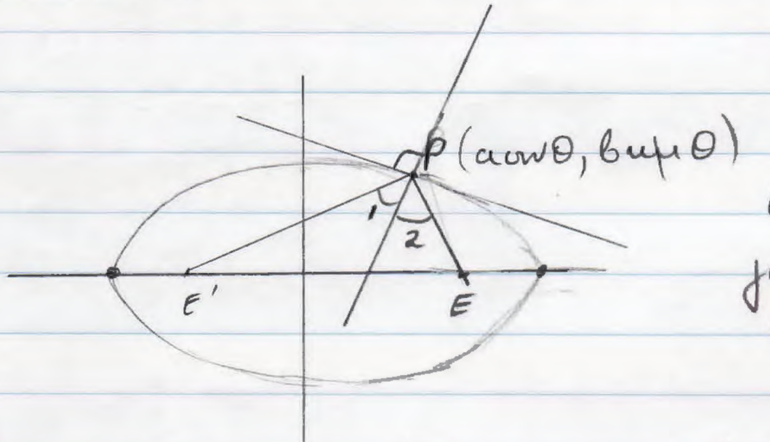
$$\Rightarrow \lambda_{E\Gamma}\cdot\lambda_{E\Delta} = \frac{b(1-\sigma\upsilon\nu\theta)}{\eta\mu\theta(a-\gamma)} \cdot \frac{b(1+\sigma\upsilon\nu\theta)}{\eta\mu\theta(-a-\gamma)} = \frac{b^2(1^2-\sigma\upsilon\nu^2\theta)}{-\eta\mu^2\theta(a-\gamma)(a+\gamma)}$$

$$= \frac{b^2\cancel{\eta\mu^2\theta}}{-\cancel{\eta\mu^2\theta}\cdot(a^2-\gamma^2)} = -\frac{b^2}{b^2} = -1 \Rightarrow E\Gamma \perp E\Delta$$

$$\Rightarrow \Gamma\hat{E}\Delta = 90^\circ$$

Ομάδα Β

1.



Ν.δ.ο $\hat{P}_1 = \hat{P}_2$

Επειδή $\hat{P}_1$ και $\hat{P}_2$ οξείες γωνίες είναι αρκετό να δείξω ότι $\varepsilon\varphi\hat{P}_1 = \varepsilon\varphi\hat{P}_2$

$$\varepsilon\varphi\hat{P}_1 = \frac{|\lambda_{E'P} - \lambda_{\kappa\alpha\theta.P}|}{1+\lambda_{E'P}\cdot\lambda_{\kappa\alpha\theta.P}} \qquad \varepsilon\varphi\hat{P}_2 = \frac{|\lambda_{EP} - \lambda_{\kappa\alpha\theta.P}|}{1+\lambda_{EP}\cdot\lambda_{\kappa\alpha\theta.P}}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \text{- - - -}\ \lambda_{\varepsilon\varphi P} = -\frac{b\sigma\upsilon\nu\theta}{a\eta\mu\theta} \Rightarrow \lambda_{KP} = \frac{a\eta\mu\theta}{b\sigma\upsilon\nu\theta}.$$

$$\Rightarrow \lambda_{E'P} = \frac{y_P - y_{E'}}{x_P - x_{E'}} = \frac{b\eta\mu\theta - 0}{a\sigma\upsilon\nu\theta - (-\gamma)} = \frac{b\eta\mu\theta}{a\sigma\upsilon\nu\theta+\gamma}$$

$$\lambda_{EP} = \frac{y_P - y_E}{x_P - x_E} = \frac{b\eta\mu\theta}{a\sigma\upsilon\nu\theta - \gamma} = \frac{b\eta\mu\theta}{a\sigma\upsilon\nu\theta - \gamma}$$

$$\Rightarrow \text{εφ}\hat{P}_1 = \frac{\left|\frac{b\,\text{ημ}\theta}{a\,\text{συν}\theta+\gamma} - \frac{a\,\text{ημ}\theta}{b\,\text{συν}\theta}\right|}{1+\frac{ab\,\text{ημ}^2\theta}{b\,\text{συν}\theta(a\,\text{συν}\theta+\gamma)}} = \frac{\left|\frac{b^2\text{ημ}\theta\,\text{συν}\theta - a^2\text{ημ}\theta\,\text{συν}\theta - a\gamma\,\text{ημ}\theta}{\cancel{b\,\text{συν}\theta(a\,\text{συν}\theta+\gamma)}}\right|}{\frac{ab\,\text{συν}^2\theta + b\gamma\,\text{συν}\theta + ab\,\text{ημ}^2\theta}{\cancel{b\,\text{συν}\theta(a\,\text{συν}\theta+\gamma)}}} =$$

$$= \frac{|\text{ημ}\theta\,\text{συν}\theta(b^2-a^2) - a\gamma\,\text{ημ}\theta|}{ab(\text{συν}^2\theta+\text{ημ}^2\theta)+b\gamma\,\text{συν}\theta} = \frac{|\text{ημ}\theta\,\text{συν}\theta(-\gamma^2) - a\gamma\,\text{ημ}\theta|}{ab+b\gamma\,\text{συν}\theta} =$$

$$= \frac{|-\gamma\,\text{ημ}\theta\,\cancel{(\gamma\,\text{συν}\theta+a)}|}{b\cancel{(a+\gamma\,\text{συν}\theta)}} = \frac{|-\gamma\,\text{ημ}\theta|}{b} = \frac{\gamma\,\text{ημ}\theta}{b} \quad (1)$$

$$\text{εφ}\hat{P}_2 = \frac{\left|\frac{b\,\text{ημ}\theta}{a\,\text{συν}\theta-\gamma} - \frac{a\,\text{ημ}\theta}{b\,\text{συν}\theta}\right|}{1+\frac{b\,\text{ημ}\theta}{(a\,\text{συν}\theta-\gamma)}\cdot\frac{a\,\text{ημ}\theta}{b\,\text{συν}\theta}} = \frac{\left|\frac{b^2\text{ημ}\theta\,\text{συν}\theta - a^2\text{ημ}\theta\,\text{συν}\theta + a\gamma\,\text{ημ}\theta}{\cancel{(b\,\text{συν}\theta)(a\,\text{συν}\theta-\gamma)}}\right|}{\frac{ab\,\text{συν}^2\theta - \gamma b\,\text{συν}\theta + ab\,\text{ημ}^2\theta}{\cancel{b\,\text{συν}\theta(a\,\text{συν}\theta-\gamma)}}}$$

$$= \frac{|\text{ημ}\theta\,\text{συν}\theta(b^2-a^2) + a\gamma\,\text{ημ}\theta|}{ab(\text{συν}^2\theta+\text{ημ}^2\theta)-\gamma b\,\text{συν}\theta} = \frac{|\text{ημ}\theta\,\text{συν}\theta(-\gamma^2) + a\gamma\,\text{ημ}\theta|}{ab-\gamma b\,\text{συν}\theta} =$$

$$= \frac{|\text{ημ}\theta\cdot\gamma\cancel{(-\gamma\,\text{συν}\theta+a)}|}{b\cancel{(a-\gamma\,\text{συν}\theta)}} = \frac{\gamma\,\text{ημ}\theta}{b} \quad (2)$$

$$\Rightarrow \text{εφ}\hat{P}_1 = \text{εφ}\hat{P}_2 \Rightarrow \boxed{\hat{P}_1 = \hat{P}_2}$$

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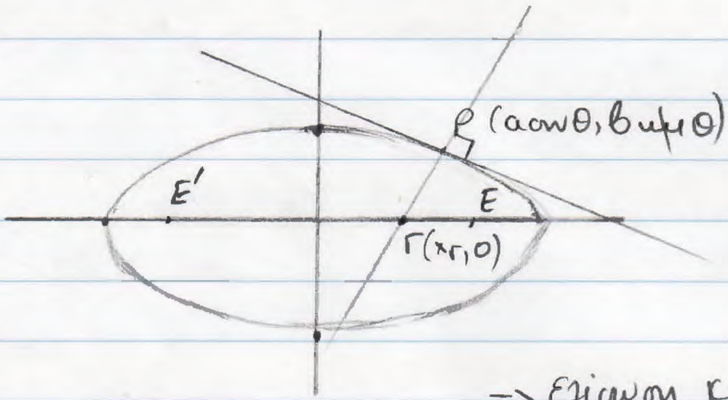
(2)



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\ldots\ldots \Rightarrow \lambda_{\text{εφ}P} = -\frac{b\,\text{συν}\theta}{a\,\text{ημ}\theta}$$

$$\Rightarrow \lambda_{\text{κ}P} = \frac{a\,\text{ημ}\theta}{b\,\text{συν}\theta}$$

$\Rightarrow$ Εξίσωση καθ. P: $y - b\,\text{ημ}\theta = \frac{a\,\text{ημ}\theta}{b\,\text{συν}\theta}(x - a\,\text{συν}\theta)$

$$\Rightarrow y b \cos\theta - b^2 \sin\theta \cos\theta = x a \sin\theta - a^2 \sin\theta \cos\theta$$

$$\Rightarrow \boxed{y b \cos\theta - x a \sin\theta = (b^2 - a^2)\sin\theta\cos\theta.}$$ εξίσωση της κάθετης στο P.

$$y_\Gamma = 0 \Rightarrow -x a \sin\theta = (b^2-a^2)\sin\theta\cos\theta \Rightarrow x_\Gamma = \frac{(b^2-a^2)\sin\theta\cos\theta}{-a\sin\theta}$$

$$\Rightarrow \boxed{x_\Gamma = \frac{(a^2-b^2)\cos\theta}{a}}$$

$$\Rightarrow (P\Gamma)^2 = \left(a\cos\theta - \frac{(a^2-b^2)\cos\theta}{a}\right)^2 + (b\sin\theta - 0)^2$$

$$= \left(\frac{\cancel{a^2\cos\theta} - \cancel{a^2\cos\theta} + b^2\cos\theta}{a}\right)^2 + b^2\sin^2\theta$$

$$= \frac{b^4\cos^2\theta}{a^2} + b^2\sin^2\theta = \frac{b^2(b^2\cos^2\theta + a^2\sin^2\theta)}{a^2}$$

$$PE = a - \varepsilon x_P = a - \frac{\gamma}{\cancel{a}}\cdot\cancel{a}\cos\theta = a - \gamma\cos\theta$$

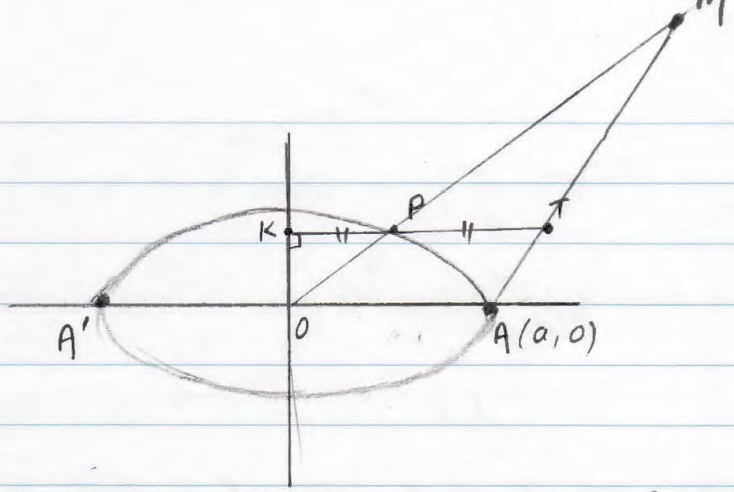
$$PE' = a + \varepsilon x_P = a + \frac{\gamma}{\cancel{a}}\cancel{a}\cos\theta = a + \gamma\cos\theta$$

$$\Rightarrow (PE)(PE') = a^2 - \gamma^2\cos^2\theta = a^2 - (a^2-b^2)\cos^2\theta$$
$$= a^2 - a^2\cos^2\theta + b^2\cos^2\theta$$

$$\Rightarrow \frac{(P\Gamma)^2}{(EP)(E'P)} = \frac{\dfrac{b^2[b^2\cos^2\theta + a^2(1-\cos^2\theta)]}{a^2}}{a^2 - a^2\cos^2\theta + b^2\cos^2\theta} = \frac{b^2(\cancel{b^2\cos^2\theta + a^2 - a^2\cos^2\theta})}{a^2(\cancel{a^2 - a^2\cos^2\theta + b^2\cos^2\theta})}$$

$$= \frac{b^2}{a^2} \quad //$$

3



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$P(a\sigma\upsilon\nu\theta, b\eta\mu\theta)$

$K(0, b\eta\mu\theta)$

$T(2a\sigma\upsilon\nu\theta, b\eta\mu\theta)$

$$\lambda_{OP}=\frac{b\eta\mu\theta-0}{a\sigma\upsilon\nu\theta-0}=\frac{b\eta\mu\theta}{a\sigma\upsilon\nu\theta}$$

$$\lambda_{AT}=\frac{b\eta\mu\theta-0}{2a\sigma\upsilon\nu\theta-a}=\frac{b\eta\mu\theta}{a(2\sigma\upsilon\nu\theta-1)}$$

$\Rightarrow$ εξίσωση της OP: $y-0=\frac{b\eta\mu\theta}{a\sigma\upsilon\nu\theta}(x-0)\Rightarrow \boxed{y=\frac{b\eta\mu\theta}{a\sigma\upsilon\nu\theta}\cdot x}$

εξίσωση της AT: $y-0=\frac{b\eta\mu\theta}{a(2\sigma\upsilon\nu\theta-1)}(x-a)\Rightarrow y=\frac{b\eta\mu\theta}{a(2\sigma\upsilon\nu\theta-1)}(x-a)$

$\Rightarrow$ Λύνω σύστημα των εξισώσεων των OP και AT.

$$\frac{b\eta\mu\theta}{a\sigma\upsilon\nu\theta}\cdot x=\frac{b\eta\mu\theta}{a(2\sigma\upsilon\nu\theta-1)}x-\frac{\cancel{a}b\eta\mu\theta}{\cancel{a}(2\sigma\upsilon\nu\theta-1)}\Rightarrow$$

$$\Rightarrow\left(\frac{b\eta\mu\theta}{a\sigma\upsilon\nu\theta}-\frac{b\eta\mu\theta}{a(2\sigma\upsilon\nu\theta-1)}\right)x=-\frac{b\eta\mu\theta}{2\sigma\upsilon\nu\theta-1}$$

$$\Rightarrow\left(\frac{2ab\eta\mu\theta\sigma\upsilon\nu\theta-ab\eta\mu\theta-ab\eta\mu\theta\sigma\upsilon\nu\theta}{a^2\sigma\upsilon\nu\theta\,\cancel{(2\sigma\upsilon\nu\theta-1)}}\right)x=-\frac{b\eta\mu\theta}{\cancel{2\sigma\upsilon\nu\theta-1}}$$

$$\Rightarrow\frac{ab\eta\mu\theta\sigma\upsilon\nu\theta-ab\eta\mu\theta}{a^2\sigma\upsilon\nu\theta}x=-\frac{b\eta\mu\theta}{1}$$

$$\Rightarrow\frac{\cancel{a}b\eta\mu\theta(\sigma\upsilon\nu\theta-1)}{a^{\cancel{2}}\sigma\upsilon\nu\theta}x=-b\eta\mu\theta\quad\Rightarrow x=-\frac{a\cancel{b\eta\mu\theta}\sigma\upsilon\nu\theta}{\cancel{b\eta\mu\theta}(\sigma\upsilon\nu\theta-1)}$$

$$\Rightarrow x = -\frac{a\,\sigma\upsilon\nu\theta}{\sigma\upsilon\nu\theta - 1} \qquad \Rightarrow y = \frac{b\,\eta\mu\theta}{\cancel{a}\,\cancel{\sigma\upsilon\nu\theta}}\cdot\left(-\frac{\cancel{a}\,\cancel{\sigma\upsilon\nu\theta}}{\sigma\upsilon\nu\theta - 1}\right)$$

$$\Rightarrow \boxed{y = -\frac{b\,\eta\mu\theta}{\sigma\upsilon\nu\theta - 1}}$$

$$\Rightarrow \frac{x}{y} = \frac{-ab\,\sigma\upsilon\nu\theta}{-b\,\eta\mu\theta} = a\,\sigma\varphi\theta. \qquad \Rightarrow \boxed{\sigma\varphi\theta = \frac{x}{ay}} \Rightarrow \boxed{\varepsilon\varphi\theta = \frac{ay}{x}}$$

$$x\,\sigma\upsilon\nu\theta - x = -ab\,\sigma\upsilon\nu\theta \Rightarrow x\,\sigma\upsilon\nu\theta + ab\,\sigma\upsilon\nu\theta = x$$

$$\Rightarrow \boxed{\sigma\upsilon\nu\theta = \frac{x}{x+ab}} \Rightarrow \tau\varepsilon\mu\,\theta = \frac{x+ab}{x}$$

$$\Rightarrow \varepsilon\varphi^2\theta + 1 = \tau\varepsilon\mu^2\theta \Rightarrow \frac{a^2y^2}{x^2} + 1 = \frac{(x+ab)^2}{x^2}$$

$$\Rightarrow a^2y^2 + x^2 = (x+ab)^2$$

$$\Rightarrow a^2y^2 + \cancel{x^2} = \cancel{x^2} + 2abx + a^2b^2$$

$$\Rightarrow \boxed{y^2 = \frac{2bx}{a} + b^2} \quad \Gamma.\text{T } \tau o\upsilon\ M.$$

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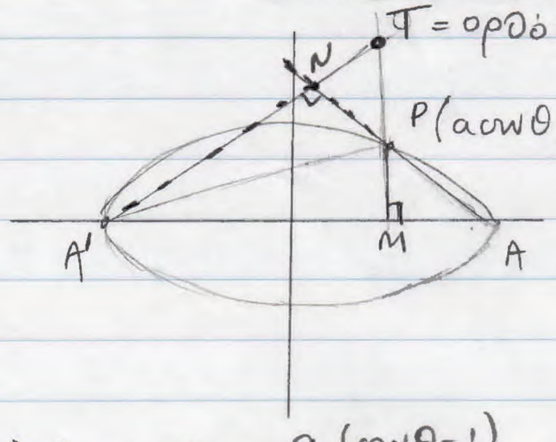
(4)



Τ = ορθόκεντρο = σημείο τομής των υψών του τριγώνου

εξίσωση ύψους PM: $\boxed{x = a\,\sigma\upsilon\nu\theta.}$ (1)

$$\lambda_{PA} = \frac{b\,\eta\mu\theta}{a\,\sigma\upsilon\nu\theta - a} = \frac{b\,\eta\mu\theta}{a(\sigma\upsilon\nu\theta - 1)}$$

$$\Rightarrow \lambda_{A'N} = -\frac{a(\sigma\upsilon\nu\theta - 1)}{b\,\eta\mu\theta}$$

$\Rightarrow$ εξίσωση του ύψους A'N : $y - 0 = -\dfrac{a(\sigma\upsilon\nu\theta - 1)}{b\,\eta\mu\theta}\left(x - (-a)\right)$

$$\Rightarrow \boxed{y = -\frac{a(\sigma\upsilon\nu\theta - 1)}{b\,\eta\mu\theta}(x+a)} \quad (2)$$

Λύνω σύστημα των (1) και (2) για να βρώ το σημείο τομής τους, "το ορθόκεντρο" Τ.

$$\Rightarrow y = -\frac{a(\sigma\upsilon\nu\theta - 1)}{b\,\eta\mu\theta}(a\,\sigma\upsilon\nu\theta + a)$$

$$\Rightarrow y = \frac{-a(\sigma\upsilon\nu\theta - 1)\cdot a(\sigma\upsilon\nu\theta + 1)}{b\,\eta\mu\theta} = -\frac{a^2(\sigma\upsilon\nu^2\theta - 1^2)}{b\,\eta\mu\theta}$$

$$\Rightarrow y = -\frac{a^2(-\eta\mu^{\cancel{2}}\theta)}{b\,\cancel{\eta\mu}\,\theta} = \frac{a^2}{b}\eta\mu\theta \Rightarrow \boxed{y_T = \frac{a^2}{b}\eta\mu\theta}$$

$$\boxed{x_T = a\,\sigma\upsilon\nu\theta}$$

$$\Rightarrow \eta\mu\theta = \frac{yb}{a^2}, \quad \sigma\upsilon\nu\theta = \frac{x}{a}$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2b^2}{a^4} = 1 \Rightarrow \boxed{\frac{x^2}{a^2} + \frac{y^2}{\left(\frac{a^2}{b}\right)^2} = 1}$$