Λύσεις ασκήσεων σελ 115:

(1) (β) $x \cdot y = 16 \Rightarrow c = 4$

$A(4,4)$, $A'(-4,4)$ Κορυφές , $x=0$, $y=0$ Ασύμπτωτες

Παραμετρικές: $\left.\begin{array}{l} x = ct \\ y = \frac{c}{t} \end{array}\right\} \Rightarrow \begin{array}{l} x = 4t \\ y = \frac{4}{t} \end{array}$, $t \in \mathbb{R} - \{0\}$.

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(2) (α) $x \cdot y = 25 \Rightarrow x = 5t$, $y = \frac{5}{t}$

(β) $y = \frac{1}{x} \Rightarrow yx = 1 \Rightarrow x = t$, $y = \frac{1}{t}$

(γ) $x \cdot y + 4 = 0 \Rightarrow xy = -4 \Rightarrow x = 2t$, $y = -\frac{2}{t}$

(δ) $36xy = 1 \Rightarrow xy = \frac{1}{36} \Rightarrow x = \frac{1}{6}t$, $y = \frac{\frac{1}{6}}{t} = \frac{1}{6t}$

$t \in \mathbb{R} - \{0\}$

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(3) (α) $\left.\begin{array}{l} x = 2t \\ y = \frac{2}{t} \Rightarrow t = \frac{2}{y} \end{array}\right\} \Rightarrow x = 2 \cdot \frac{2}{y} \Rightarrow \boxed{xy = 4}$

(β) $\left.\begin{array}{l} x = 3t \\ y = -\frac{3}{t} \Rightarrow t = -\frac{3}{y} \end{array}\right\} \Rightarrow x = 3 \cdot \left(-\frac{3}{y}\right) \Rightarrow \boxed{xy = -9}$

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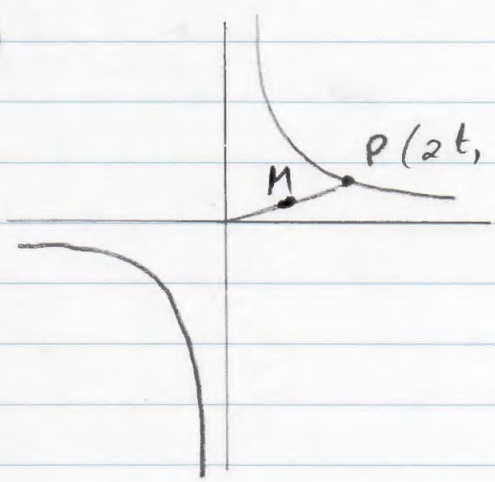
(4)

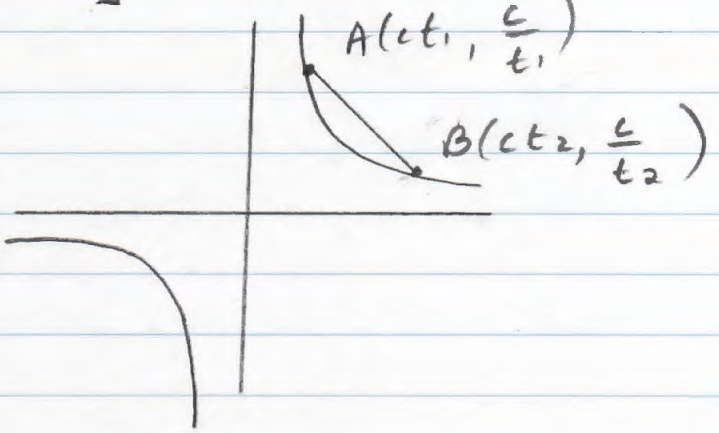


$xy = 4$

$x_M = \frac{0+2t}{2} = t$, $y_M = \frac{0+\frac{2}{t}}{2} = \frac{1}{t}$

$\Rightarrow \left.\begin{array}{l} x = t \\ y = \frac{1}{t} \end{array}\right\} \Rightarrow y = \frac{1}{x} \Rightarrow \boxed{yx = 1}$

(5) $x \cdot y = c^2$



$$\lambda_{AB} = \frac{\frac{c}{t_1} - \frac{c}{t_2}}{ct_1 - ct_2} =$$

$$= \frac{\frac{ct_2 - ct_1}{t_1 t_2}}{ct_1 - ct_2} =$$

$$= \frac{-(ct_1 - ct_2)}{t_1 t_2 (ct_1 - ct_2)} = -\frac{1}{t_1 t_2}$$

$$\Rightarrow y - \frac{c}{t_1} = -\frac{1}{t_1 t_2}(x - ct_1)$$

$$\Rightarrow y t_1 t_2 - t_2 c = -x + ct_1 \Rightarrow \boxed{x + y t_1 t_2 = ct_1 + ct_2}$$

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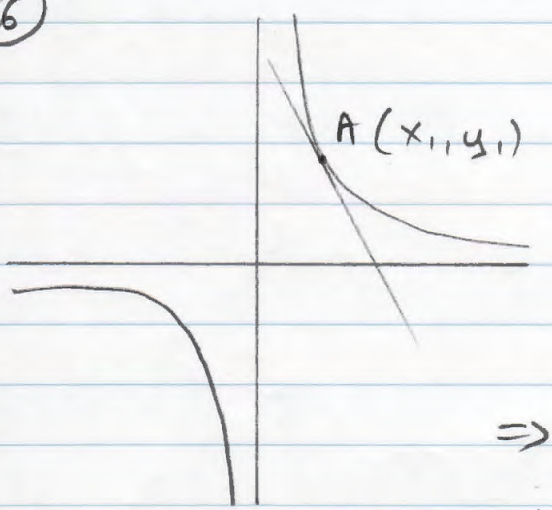
(6)



$$x \cdot y = c^2 \Rightarrow 1 \cdot y + x y' = 0$$

$$\Rightarrow x y' = -y \Rightarrow \boxed{y' = -\frac{y}{x}}$$

$$\Rightarrow \boxed{\lambda_{\varepsilon\varphi A} = -\frac{y_1}{x_1}}$$

$$\Rightarrow y - y_1 = -\frac{y_1}{x_1}(x - x_1) \Rightarrow y x_1 - y_1 x_1 = -y_1 x + y_1 x_1$$

$$\Rightarrow y_1 x + y x_1 = 2 y_1 x_1 \quad (\text{αφού } x \cdot y = c^2)$$

$$\Rightarrow \boxed{y_1 x + y x_1 = 2c^2}$$

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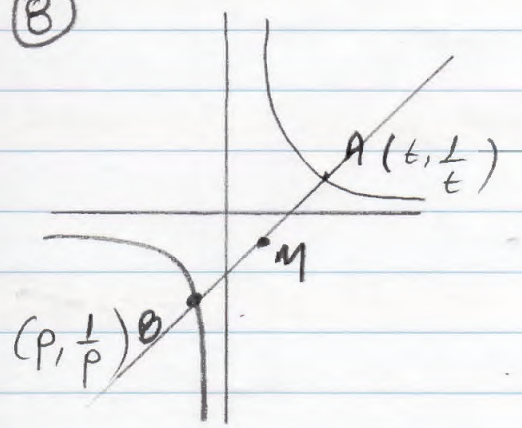
(7) ευθύς όμως

(8)



$xy = 1 \qquad \lambda_{AB} = 1$

$$\lambda_{AB} = \frac{\frac{1}{t} - \frac{1}{\rho}}{t - \rho} = \frac{\frac{\rho - t}{t\rho}}{t - \rho} = -\frac{1}{t\rho} \Bigg\} \Rightarrow$$

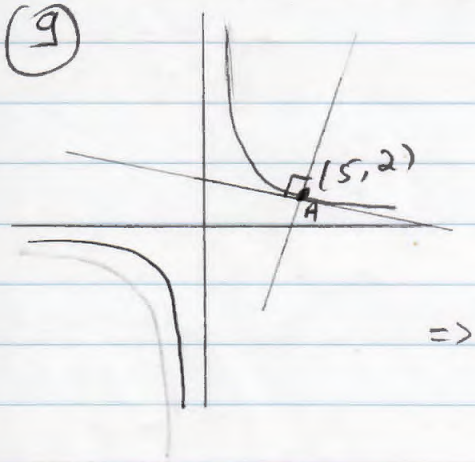
$$\lambda_{AB} = 1$$

$$\Rightarrow -\frac{1}{t\rho} = 1 \Rightarrow \boxed{t\rho = -1}$$

$$x_M = \frac{t+\rho}{2} \Rightarrow t + \rho = 2x$$

$$y_M = \frac{\rho + \frac{1}{t}}{2} = \frac{t+\rho}{2\rho t} \Bigg\} \Rightarrow y = \frac{2x}{2\rho t} = \frac{x}{-1} \Rightarrow \boxed{y = -x}$$

(9)



$xy = 10 \Rightarrow y + xy' = 0 \Rightarrow y' = -\frac{y}{x}$

$\Rightarrow \lambda_{\varepsilon\varphi_A} = -\frac{2}{5} \Rightarrow \lambda_{\kappa_A} = \frac{5}{2}$

$\Rightarrow$ Εξίσωση εφαπτόμενης: $y - 2 = -\frac{2}{5}(x-5) \Rightarrow$

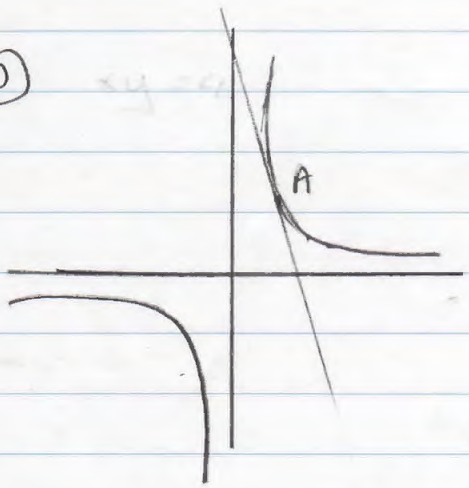
$\Rightarrow 5y - 10 = -2x + 10$

$\Rightarrow \boxed{2x + 5y = 20}$

Εξίσωση κάθετης:

$y - 2 = \frac{5}{2}(x-5) \Rightarrow 2y - 4 = 5x - 25 \Rightarrow \boxed{5x - 2y = 21}$

(10)



$xy=4 \Rightarrow y+xy'=0 \Rightarrow y'=-\frac{y}{x}$

1ος τρόπος

@ $\left.\begin{array}{l} \lambda_{\varepsilon\varphi}=-\frac{1}{2} \\ \lambda_{\varepsilon\varphi}=-\frac{y}{x} \end{array}\right\} \Rightarrow \frac{y}{x}=\frac{1}{2} \Rightarrow \boxed{x_A=2y_A}$

Όμως $x_A \cdot y_A=4 \Rightarrow 2y_A \cdot y_A=4 \Rightarrow \boxed{y_A^2=2}$

$\Rightarrow y_A=\pm\sqrt{2} \Rightarrow x_A=\pm 2\sqrt{2}$

$\Rightarrow y-\sqrt{2}=\frac{-1}{2}(x-2\sqrt{2}) \Rightarrow 2y-2\sqrt{2}=-x+2\sqrt{2} \Rightarrow \boxed{2y+x=4\sqrt{2}}$

$y-(-\sqrt{2})=-\frac{1}{2}\left(x-(-2\sqrt{2})\right) = 2y+2\sqrt{2}=-x-2\sqrt{2} \Rightarrow \boxed{2y+x=-4\sqrt{2}}$

2ος τρόπος

$\left.\begin{array}{l} xy=4 \\ y=-\frac{1}{2}x+b \end{array}\right\} \Rightarrow x\left(-\frac{1}{2}x+b\right)=4 \Rightarrow -x^2+2xb=8$

$\Rightarrow x^2-2xb+8=0$

$\Delta=0$

$\Rightarrow 4b^2-4\cdot 1\cdot 8=0 \Rightarrow 4b^2=32 \Rightarrow b^2=8 \Rightarrow b=\pm\sqrt{8}=\pm 2\sqrt{2}$

$\Rightarrow \boxed{y=\frac{-1}{2}x+2\sqrt{2}}$ ή $\boxed{y=\frac{-1}{2}x-2\sqrt{2}}$

(β) $\left.\begin{array}{l} y=\lambda x+b \\ x\cdot y=4 \end{array}\right\} x\cdot(\lambda x+b)=4 \Rightarrow \left.\begin{array}{l} \lambda x^2+bx-4=0 \\ \Delta=0 \end{array}\right\} \Rightarrow$

$\Rightarrow \boxed{b^2+16\lambda=0}$ (1) Όμως $(3,-4)$ σημείο της $y=\lambda x+b$

$\Rightarrow \boxed{-4=3\lambda+b}$ (2) $\Rightarrow \lambda=\frac{-b-4}{3}$

$\Rightarrow$ από (1)+(2) $\Rightarrow b^2+16\left(\frac{-b-4}{3}\right)=0$

$\Rightarrow 3b^2 - 16b - 64 = 0 \Rightarrow b_{1,2} = \frac{16 \pm \sqrt{256+768}}{6} = \frac{16 \pm \sqrt{1024}}{6}$

$= \frac{16 \pm 32}{6} \begin{cases} b_1 = 8 \\ b_2 = -\frac{16}{6} = -\frac{8}{3} \end{cases}$

$\Rightarrow$ αν $b_1 = 8 \Rightarrow \lambda_1 = \frac{-8-4}{3} = \frac{-12}{3} = -4 \Rightarrow \boxed{y = -4x + 8}$

αν $b_2 = -\frac{8}{3} \Rightarrow \lambda_2 = \frac{-\left(-\frac{8}{3}\right)-4}{3} = \frac{+\frac{8}{3}-12}{3} = \frac{-4}{9} \Rightarrow \boxed{y = -\frac{4}{9}x - \frac{8}{3}}$

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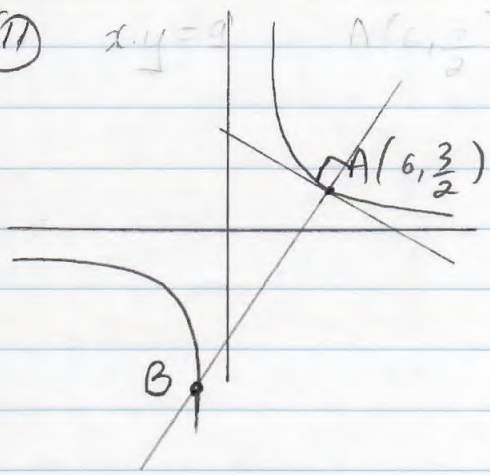
(11)



$x \cdot y = 9 \Rightarrow y + xy' = 0 \Rightarrow y' = -\frac{y}{x}$

$\Rightarrow \lambda_{\varepsilon\varphi A} = -\frac{\frac{3}{2}}{6} = -\frac{3}{12} = -\frac{1}{4} \Rightarrow \boxed{\lambda_K = 4}$

$\Rightarrow$ εξίσωση AB: $y - \frac{3}{2} = 4(x-6)$

$\Rightarrow 2y - 3 = 8x - 48 \Rightarrow \boxed{8x - 2y = 45}$

Λύνω σύστημα της εξίσωσης της υπερβολής και της AB για να βρώ τα κοινά σημεία. (Αναμένεται να βρώ το A και ένα δεύτερο το οποίο θα είναι το B.)

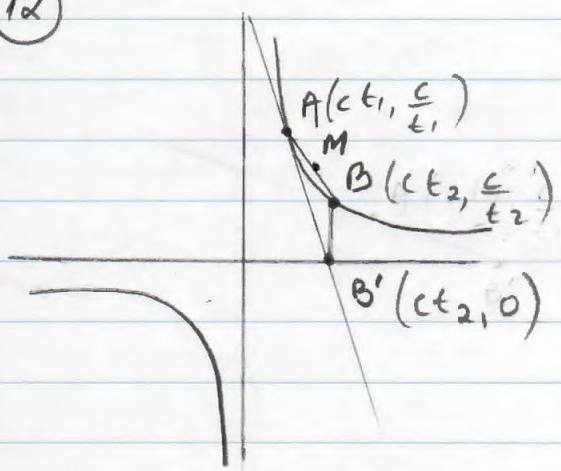
$\left.\begin{matrix} x \cdot y = 9 \Rightarrow x = \frac{9}{y} \\ 8x - 2y = 45 \end{matrix}\right\} \Rightarrow 8 \cdot \frac{9}{y} - 2y = 45 \Rightarrow 72 - 2y^2 = 45y$

$\Rightarrow 2y^2 + 45y - 72 = 0 \Rightarrow y_{1,2} = \frac{-45 \pm \sqrt{2025 - 4 \cdot 2 \cdot (-72)}}{2 \cdot 2} =$

$= \frac{-45 \pm \sqrt{2025+576}}{4} = \frac{-45 \pm \sqrt{2601}}{4} = \frac{-45 \pm 51}{4} = \begin{cases} y_A = \frac{3}{2} \\ y_B = -24 \end{cases}$

$\Rightarrow x_B = \frac{9}{-24} = -\frac{3}{8} \Rightarrow \boxed{B\left(-\frac{3}{8}, -24\right)}$

(12)



$x \cdot y = c^2$

$x_M = \frac{ct_1 + ct_2}{2} = \frac{c(t_1+t_2)}{2}$

$y_M = \frac{\frac{c}{t_1} + \frac{c}{t_2}}{2} = \frac{c(t_2+t_1)}{2t_1t_2}$

Όμως $\left.\begin{aligned} \lambda_{\varepsilon\varphi A} &= -\frac{y}{x} = -\frac{\frac{c}{t_1}}{ct_1} = -\frac{1}{t_1^2} \\ \lambda_{AB'} &= \frac{\frac{c}{t_1} - 0}{ct_1 - ct_2} = \frac{c\,1}{c(t_1-t_2)t_1} \end{aligned}\right\} \Rightarrow -\frac{1}{t_1^2} = \frac{1}{t_1^2 - t_1t_2}$

$\Rightarrow t_1^2 - t_1t_2 = -t_1^2$

$\Rightarrow 2t_1^2 - t_1t_2 = 0$

$\Rightarrow t_1(2t_1 - t_2) = 0 \Rightarrow t_1 = 0$ αδύν.

$\Rightarrow \boxed{2t_1 = t_2}$

$\Rightarrow x = \frac{3t_1 \cdot c}{2}$

$y = \frac{3t_1 \cdot c}{4t_1^2} \Rightarrow y = \frac{3c}{4t_1} \Rightarrow t_1 = \frac{3c}{4y}$

$\Rightarrow x = \frac{3 \cdot \frac{3c}{4y} \cdot c}{2}$

$\Rightarrow x = \frac{9c^2}{8y}$

$\Rightarrow \boxed{xy = \frac{9}{8}c^2}$ ισοσκελής υπερβολή

με $\boxed{c' = \frac{3}{\sqrt{8}}c}$