1

1) **Ρητοποίηση:** $\dfrac{\beta}{\sqrt[\nu]{\alpha^{\kappa}}}\cdot\dfrac{\sqrt[\nu]{\alpha^{\nu-\kappa}}}{\sqrt[\nu]{\alpha^{\nu-\kappa}}}$ $\quad$ $\dfrac{\beta}{(\sqrt{\alpha}\pm\gamma)}\cdot\dfrac{(\sqrt{\alpha}\mp\gamma)}{(\sqrt{\alpha}\mp\gamma)}$

α) $\dfrac{7}{\sqrt{5}}$, β) $\dfrac{10}{\sqrt[3]{5}}$, γ) $\dfrac{2}{\sqrt[5]{2^4}}$, δ) $\dfrac{6}{\sqrt[4]{3^3}}$

ε) $\dfrac{8}{\sqrt{2}+1}$, ζ) $\dfrac{5}{\sqrt{5}-2}$, η) $\dfrac{7}{\sqrt{7}-\sqrt{3}}$, θ) $\dfrac{1}{3+\sqrt{2}}$

ι) $\dfrac{1}{\sqrt[10]{2^6}}$, κ) $\dfrac{4}{\sqrt[8]{7^5}}$, λ) $\dfrac{\sqrt{2}+1}{\sqrt{3}-1}$, μ) $\dfrac{3+\sqrt{5}}{2+\sqrt{5}}$

2) **Ρητός εκθέτης** $\sqrt[\nu]{\alpha^{\kappa}} = \alpha^{\frac{\kappa}{\nu}}$, $\alpha \geq 0$

α) $\sqrt[3]{\sqrt{3}}$, β) $\sqrt[3]{2}\cdot\sqrt[4]{2}$, γ) $\sqrt{3}+\sqrt{12}$

δ) $x^{\frac{1}{2}}\cdot x^{\frac{2}{3}} - \dfrac{\sqrt{x^3}\cdot\sqrt[3]{x^2}}{x}$, $x>0$

Λύσεις:

1α) $\dfrac{7}{\sqrt{5}}$, $\nu=2,\ \kappa=1$, $\nu-\kappa=1$ $\Big\} \Rightarrow \dfrac{7}{\sqrt[2]{5}} \cdot \dfrac{\sqrt[2]{5^1}}{\sqrt[2]{5^1}} = \dfrac{7\sqrt{5}}{5}$

β) $\dfrac{10}{\sqrt[3]{5}}$, $\nu=3,\ \kappa=1$, $\nu-\kappa=2$ $\Big\} \Rightarrow \dfrac{10}{\sqrt[3]{5}} \cdot \dfrac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}} = \dfrac{\cancel{10}^{2}\sqrt[3]{5^2}}{\cancel{5}_{1}} = 2\sqrt[3]{5^2}$

γ) $\dfrac{2}{\sqrt[5]{2^4}}$, $\nu=5,\ \kappa=4$, $\nu-\kappa=1$ $\Big\} \Rightarrow \dfrac{2}{\sqrt[5]{2^4}} \cdot \dfrac{\sqrt[5]{2^1}}{\sqrt[5]{2^1}} = \dfrac{\cancel{2}^{1}\sqrt[5]{2}}{\cancel{2}_{1}} = \sqrt[5]{2}$

δ) $\dfrac{6}{\sqrt[4]{3^3}}$, $\nu=4,\ \kappa=3$, $\nu-\kappa=1$ $\Big\} \Rightarrow \dfrac{6}{\sqrt[4]{3^3}} \cdot \dfrac{\sqrt[4]{3^1}}{\sqrt[4]{3^1}} = \dfrac{\cancel{6}^{2}\sqrt[4]{3}}{\cancel{3}_{1}} = 2\sqrt[4]{3}$

ε) $\dfrac{8}{(\sqrt{2}+1)} \cdot \dfrac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = \dfrac{8(\sqrt{2}-1)}{(\sqrt{2})^2-1^2} = \dfrac{8(\sqrt{2}-1)}{2-1} = 8(\sqrt{2}-1)$

ϛ) $\dfrac{5}{(\sqrt{5}-2)} \cdot \dfrac{(\sqrt{5}+2)}{(\sqrt{5}+2)} = \dfrac{5(\sqrt{5}+2)}{5-4} = 5(\sqrt{5}+2)$

η) $\dfrac{7}{(\sqrt{7}-\sqrt{3})} \cdot \dfrac{(\sqrt{7}+\sqrt{3})}{(\sqrt{7}+\sqrt{3})} = \dfrac{7(\sqrt{7}+\sqrt{3})}{7-3} = \dfrac{7(\sqrt{7}+\sqrt{3})}{4}$

θ) $\dfrac{1}{(3+\sqrt{2})} \cdot \dfrac{(3-\sqrt{2})}{(3-\sqrt{2})} = \dfrac{3-\sqrt{2}}{3-2} = 3-\sqrt{2}$

ι) $\dfrac{1}{\sqrt[\cancel{10}\,5]{2^{\cancel{6}\,3}}} = \dfrac{1}{\sqrt[5]{2^3}}$, $\nu=5,\ \kappa=3$, $\nu-\kappa=2$ $\Big\} \Rightarrow \dfrac{1}{\sqrt[5]{2^3}} \cdot \dfrac{\sqrt[5]{2^2}}{\sqrt[5]{2^2}} = \dfrac{\sqrt[5]{2^2}}{2}$

κ) $\dfrac{4}{\sqrt[8]{7^5}}$ $\nu=8,\ \kappa=5$, $\nu-\kappa=3$ $\Big\} \Rightarrow \dfrac{4}{\sqrt[8]{7^5}} \cdot \dfrac{\sqrt[8]{7^3}}{\sqrt[8]{7^3}} = \dfrac{4\sqrt[8]{7^3}}{7}$

ζ) $\dfrac{(\sqrt{2}+1)}{(\sqrt{3}-1)}\cdot\dfrac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = \dfrac{(\sqrt{2}+1)(\sqrt{3}+1)}{3-1} = \dfrac{\sqrt{6}+\sqrt{3}+\sqrt{2}+1}{2}$

η) $\dfrac{(3+\sqrt{5})}{(2+\sqrt{5})}\cdot\dfrac{(2-\sqrt{5})}{(2-\sqrt{5})} = \dfrac{(3+\sqrt{5})(2-\sqrt{5})}{4-5} = -(6-3\sqrt{5}+2\sqrt{5}-5)$

$= -(1-\sqrt{5}) = \sqrt{5}-1$

2α) $\sqrt[3]{\sqrt{3}} = \sqrt[3]{3^{\frac{1}{2}}} = \left(3^{\frac{1}{2}}\right)^{\frac{1}{3}} = 3^{\frac{1}{2}\cdot\frac{1}{3}} = 3^{\frac{1}{6}} = \sqrt[6]{3}$

β) $\sqrt[3]{2}\cdot\sqrt[4]{2} = 2^{\frac{1}{3}}\cdot 2^{\frac{1}{4}} = 2^{\frac{1}{2}+\frac{1}{3}} = 2^{\frac{5}{6}} = \sqrt[6]{2^5}$

γ) $\sqrt{3}+\sqrt{12} = 3^{\frac{1}{2}}+12^{\frac{1}{2}} = 3^{\frac{1}{2}}+(3\cdot 4)^{\frac{1}{2}} = 3^{\frac{1}{2}}+3^{\frac{1}{2}}\cdot 4^{\frac{1}{2}}$

$= 3^{\frac{1}{2}}(1+\sqrt{4}) = \sqrt{3}(1+2) = 3\sqrt{3}$

δ) $x^{\frac{1}{2}}\cdot x^{\frac{2}{3}} - \dfrac{\sqrt{x^3}\cdot\sqrt[3]{x^2}}{x} = x^{\frac{1}{2}}\cdot x^{\frac{2}{3}} - \dfrac{x^{\frac{3}{2}}\cdot x^{\frac{2}{3}}}{x^1}$

$= x^{\frac{1}{2}+\frac{2}{3}} - x^{\frac{3}{2}+\frac{2}{3}-1} = x^{\frac{7}{6}} - x^{\frac{9+4-6}{6}} = \cancel{x^{\frac{7}{6}}} - \cancel{x^{\frac{7}{6}}} = 0$