

Ολοκλήρωση με αντικατάσταση

Τα ολοκληρώματα της μορφής $\int f(g(x))g'(x)dx$ υπολογίζονται κάνοντας την αντικατάσταση $g(x) = t$ (οπότε $g'(x)dx = dt$). Έτσι προκύπτει ένα ολοκλήρωμα της μορφής $\int f(t)dt$ που είναι δυνατόν να υπολογίζεται εύκολα. Μετά τον υπολογισμό του ολοκληρώματος $\int f(t)dt$ αντικαθιστούμε την μεταβλητή t με $g(x)$.

A. Να υπολογισθούν τα ολοκληρώματα:

$$1) \quad I_1 = \int \eta\mu(2x+3)dx \qquad 2) \quad I_2 = \int \sigma\upsilon\nu 3xdt \qquad 3) \quad I_3 = \int (3x^2+1)^4 xdx$$

$$4) \quad I_4 = \int \sqrt{\eta\mu x} \sigma\upsilon\nu x dx \qquad 5) \quad I_1 = \int e^{2x+1} dx \qquad 6) \quad I_2 = \int \frac{dx}{(x-3)^2}$$

$$7) \quad I_3 = \int \frac{dx}{x-3} \qquad 8) \quad I_4 = \int x \sigma\upsilon\nu(x^2+2)dx \qquad 9) \quad \int (5x+1)^5 dx$$

$$10) \quad \int \varepsilon\phi x dx \qquad 11) \quad \int \frac{2x+1}{x^2+x+1} \qquad 12) \quad \int \frac{xdx}{\sqrt{1+x^2}}$$

$$13) \quad \int \frac{\eta\mu x}{\sigma\upsilon\nu^3 x} dx \qquad 14) \quad \int \sqrt{x^2+1} xdx \qquad 15) \quad \int \frac{\ln x}{x} dx$$

$$16) \quad \int \frac{e^{3x}}{e^{3x}+1} dx$$

A. Να υπολογισθούν τα ολοκληρώματα:

1. $\int x\eta\mu(x^2+2) dx$

2) $\int \frac{2x-5}{x^2-5x+6} dx$

3) $\int \sigma\upsilon\nu^2 x dx$

4) $\int \sigma\upsilon\nu^3 x dx$

5) $\int \eta\mu^7 x \sigma\upsilon\nu^3 x dx$

6) $\int \frac{\varepsilon\phi^3 x}{\sigma\upsilon\nu^2 x} dx$

7) $I_1 = \int (x^3+2x^2+5)(3x^2+4x) dx$

8) $I_2 = \int \sqrt[3]{\eta\mu x} \sigma\upsilon\nu x dx$

9) $I_3 = \int \frac{x^3 + \ln x}{x} dx$

10) $\int 3^{\sqrt{x}} \frac{1}{\sqrt{x}} dx$

11) $\int \frac{3x dx}{x^2+2}$

12) $\int \frac{x - \sqrt{\tau\omicron\xi\varepsilon\phi x}}{1+x^2} dx$

13) $\int x^3 (2-3x^4)^3 dx$

14) $\int \frac{1+\ln x}{4+\ln x} dx$

15) $\int \frac{\eta\mu 4x}{\sigma\upsilon\nu^4 2x+4} dx$

16) $\int \frac{e^x}{e^x-1} dx$

17) $\int \frac{e^{1/x}}{x^2} dx$

18) $\int e^{-(x^2+1)} x dx$

19) $\int \frac{\tau\omicron\xi\varepsilon\phi \frac{x}{2}}{4+x^2} dx$

20) $\int \frac{\sigma\upsilon\nu \sqrt{x} dx}{\sqrt{x}}$

21) $\int \frac{e^t dt}{1-e^{2t}}$

22) $\int \frac{dx}{2^x+3}$

23) $\int e^x \sqrt{\alpha - \beta e^x} dx$

24) $\int \frac{a^{2x}-1}{\sqrt{a^x}} dx$

25) $\int \frac{\beta d\psi}{\sqrt{1-\psi}}$

26) $\int \frac{dx}{(\alpha+\beta)-(\alpha-\beta)x^2}$

27) $\int \frac{dx}{1+\sigma\upsilon\nu^2 x}$

28) $\int \frac{dx}{x \ln^2 x}$

29) $\int \alpha^{\eta\mu x} \sigma\upsilon\nu x dx$

30) $\int \frac{\sqrt[3]{1+\ln x}}{x} dx$

31) $\int \frac{dx}{7x^2+5}$

B. Να υπολογισθούν τα ολοκληρώματα:

$$32) \int \frac{x^2}{x^2-2} dx$$

$$33) \int \frac{e^{\tau\phi x} + \ln(1+x^2) + 1}{1+x^2} dx$$

$$34) \int \sigma\phi^2(\sigma x) dx$$

$$35) \int \frac{\alpha^x dx}{1+a^{2x}}$$

$$36) \int x^5 \sqrt{5-x^2} dx$$

$$37) \int \frac{\alpha^x - \beta^x}{\alpha^x \beta^x} dx$$

$$38) \int \frac{(\alpha^x - \beta^x)^2}{\alpha^x \beta^x} dx$$

$$39) \int \eta\mu^3 6x \cdot \sigma\nu\nu 6x dx$$

$$40) \int \frac{\eta\mu x \cdot \sigma\nu\nu x}{\sqrt{\sigma\nu\nu^2 x - \eta\mu^2 x}} dx$$

$$41) \int \varepsilon\phi\sqrt{x-1} \frac{dx}{\sqrt{x-1}}$$

$$42) \int \frac{dx}{x(4-\ln^2 x)}$$

$$43) \int \varepsilon\phi^3\left(\frac{x}{3}\right) \tau\epsilon\mu^2\left(\frac{x}{3}\right) dx$$

$$44) \int \frac{\tau\epsilon\mu^2 x dx}{\sqrt{4-\varepsilon\phi^2 x}}$$

$$45) \int \frac{2x+3}{2x+1} dx$$

$$46) \int e^{-\varepsilon\phi x} \tau\epsilon\mu^2 x dx$$

$$47) \int \frac{x^2}{\sqrt[3]{x^3+1}} dx$$

$$48) \int \frac{\left(1-\eta\mu\frac{x}{\sqrt{2}}\right)^2}{\eta\mu\frac{x}{\sqrt{2}}} dx$$

$$49) \int \frac{1-\eta\mu x}{x+\sigma\nu\nu x} dx$$

$$50) \int \varepsilon\phi^3 x dx$$

$$51) \int \sigma\nu\nu^2 x \eta\mu x dx$$

$$52) \int \frac{\eta\mu\sqrt{x}}{\sqrt{x}} dx$$

$$53) \int \frac{\eta\mu x}{1-\sigma\nu\nu x} dx$$

$$54) \int \frac{\sigma\nu\nu(\ln x)}{x} dx$$

$$55) I = \int \frac{2\sigma\nu\nu x + 3\eta\mu x}{\eta\mu^3 x} dx$$

$$56) I = \int (\sigma\phi 2x - \sigma\tau\epsilon\mu 2x)^2 dx$$

$$57) I = \int \varepsilon\phi^5 x \sigma\tau\epsilon\mu^2 x dx$$

$$58) \int \sigma\nu\nu^3 x dx$$

$$59) \int \sigma\nu\nu^4 x dx$$

$$60) \int \frac{2x}{x^2+1} dx$$

$$61) \int \frac{x}{4-x^2} dx$$

$$62) \int \frac{dx}{1+e^x}$$

$$63) \int x^2 e^{x^3} dx$$

$$64) \int (x^3-7)^8 3x^2 dx$$

$$65) \int \sqrt{\frac{\tau\phi\xi\eta\mu x}{1-x^2}} dx$$

Γ. Να υπολογισθούν τα ολοκληρώματα:

$$66) \quad I = \int \frac{2^x dx}{\sqrt{1-4^x}}$$

$$67) \quad I = \int \frac{dx}{(1+x^2)\tauοξ\epsilon\phi x}$$

$$68) \quad I = \int \frac{(\ln x)^3}{x} dx$$

$$69) \quad I = \int \frac{2x - \sqrt{\tauοξ\eta\mu x}}{\sqrt{1-x^2}} dx$$

$$70) \quad I = \int \frac{x + (\tauοξ\sigma\upsilon\nu 3x)^2}{\sqrt{1-9x^2}} dx$$

$$71) \quad I = \int \frac{1+x-x^2}{\sqrt{(1-x^2)^3}} dx$$

$$72) \quad I = \int \sqrt{\frac{1-x}{1+x}} dx$$

$$73) \quad \int \frac{x}{x+4} dx$$

$$74) \quad \int \frac{2x-1}{x-2} dx$$

$$75) \quad \int \frac{x^4 dx}{x^2+1}$$

$$76) \quad \int \frac{x^2-1}{x^2+1} dx$$

$$77) \quad \int \frac{dx}{x(x-1)}$$

$$78) \quad \int \frac{dx}{\sqrt{1-(2x+3)^2}}$$

$$79) \quad \int \frac{\eta\mu^3 x}{\sqrt{\sigma\upsilon\nu x}} dx$$

$$80) \quad \int (\epsilon\phi^2 x + \epsilon\phi^4 x) dx$$

$$81) \quad \int \tauοξ\eta\mu x dx$$

$$82) \quad \int x\sigma\upsilon\nu 3x dx$$

$$83) \quad \int \frac{x}{e^x} dx$$

$$84) \quad \int x - 3^{-x} dx$$

$$85) \quad \int (x^2 - 2x + 5)e^{-x} dx$$

$$86) \quad \int \frac{\ln x}{x^3} dx$$

$$87) \quad \int \frac{xe^x}{(1+x)^2} dx$$

$$88) \quad \int \frac{x dx}{\sigma\upsilon\nu^2 x}$$

$$89) \quad \int \eta\mu(\ln x) dx$$

$$90) \quad \int \ln(x + \sqrt{1+x^2}) dx$$

$$91) \quad \int \frac{x\tauοξ\eta\mu x}{\sqrt{1-x^2}} dx$$

$$92) \quad \int \frac{\ln(\ln x)}{x} dx$$

$$93) \quad I = \int e^{2\tauοξ\sigma\upsilon\nu x} dx$$

$$94) \quad I = \int \frac{x^2 dx}{(1+x^2)^2}$$

$$95) \quad I = \int \frac{x\tauοξ\epsilon\phi x}{\sqrt{1+x^2}} dx$$

$$96) \quad I = \int \frac{\log x}{x^3} dx$$

$$97) \quad \int x\epsilon\phi^2 x dx$$

$$98) \quad I = \int \frac{x^2 dx}{\sqrt{1-x^2}}$$

$$99) \quad I = \int \frac{x^2 e^x dx}{(x+2)^2}$$

$$100) \quad I = \int \frac{\tauοξ\eta\mu x}{\sqrt{x+1}} dx$$

$$101) \quad I = \int x^2 a^x dx$$

$$102) \quad \int \frac{x^3 dx}{\sqrt{1+x^2}}$$

Δ. Να υπολογισθούν τα ολοκληρώματα:

$$102) \quad I = \int (\operatorname{tox}\epsilon\phi x)^2 x dx$$

$$103) \quad I = \int (\operatorname{tox}\eta\mu x)^2 dx$$

$$104) \quad I = \int \frac{\operatorname{tox}\eta\mu\sqrt{x}}{\sqrt{1-x}} dx$$

$$105) \quad I = \int x^2 \ln(1+x) dx$$

$$106) \quad \int (x+1)e^{2x} dx$$

$$107) \quad \int x^3 e^{2x} dx$$

$$108) \quad \int \eta\mu x \eta\mu 3x dx$$

$$109) \quad \int \ln(x^2 + 2) dx$$

$$110) \quad \int \frac{x^2 dx}{(1-x^2)^3}$$

$$111) \quad I_1 = \int \eta\mu^{\nu-1} x \sigma\nu \nu [(\nu+1)x] dx$$

$$112) \quad I_2 = \int \eta\mu^{\nu-1} x \eta\mu [(\nu+1)x] dx$$

$$113) \quad \int \frac{dx}{(1-x^2)^3}$$

$$114) \quad I_3 = \int \sigma\nu \nu^{\nu-1} x \sigma\nu \nu [(\nu+1)x] dx$$

$$115) \quad I_4 = \int \sigma\nu \nu^{\nu-1} x \sigma\nu \nu [(\nu+1)x] dx \quad \nu \in \mathbb{N}^*$$

$$116) \quad I = \int (3x^2 + 6x + 5) \operatorname{tox}\epsilon\phi x dx$$

$$117) \quad I = \int x \ln\left(1 + \frac{1}{x}\right) dx$$

$$118) \quad I = \int \frac{\sqrt{x^2+1} [\ln(x^2+1) - 2 \ln x]}{x^4} dx$$

$$119) \quad I = \int \ln(\sqrt{1-x} + \sqrt{1+x}) dx$$

$$120) \quad I = \int (x^2 - 2x + 5) e^{-x} dx$$

$$121) \quad I = \int (3x^3 - 17) e^{2x} dx$$

$$122) \quad I = \int e^{5x} \sigma\nu \nu 4x dx$$

$$123) \quad I = \int (2x^3 + 3x^2 - 8x + 1) \sqrt{2x+6} dx$$

Ε. Να υπολογισθεί τα ολοκληρώματα:

$$I = \int (x^2 + 3x + 5) \sigma\nu \nu 2x dx$$