ΕΞΕΤΑΣΕΙΣ ΓΙΑ ΤΑ ΑΝΩΤΕΡΑ ΚΑΙ ΑΝΩΤΑΤΑ ΕΚΠΑΙΔΕΥΤΙΚΑ ΙΔΡΥΜΑΤΑ

ΜΑΘΗΜΑΤΙΚΑ (Για απόφοιτους Τεχνικών Σχολών)

ΛΥΣΕΙΣ ΔΟΚΙΜΙΟΥ 2005

ΜΕΡΟΣ Α΄

1	$\lim\limits_{x\to 0} \dfrac{e^x + x - 1}{x} = \left(\dfrac{0}{0}\right)$ απροσδιοριστία (D.L.H.) $\lim\limits_{x\to 0} \dfrac{e^x + 1}{1} = \dfrac{1+1}{1} = 2$
2	α) Θ Α Λ Α Σ Σ Α αναγραμματισμοί: $M_7^{\varepsilon} = \dfrac{7!}{3!2!} = 420$ β) Τρία Α μαζί: $M_5^{\varepsilon} = \dfrac{5!}{2!} = 60$
3	α) $\int\left(5x^3 + \dfrac{2}{x^2} + 1\right) dx = \dfrac{5x^4}{4} - \dfrac{2}{x} + x + c$ β) $\int \eta\mu^5 x \,\sigma\upsilon\nu\, x\, dx = \int \eta\mu^5 x \left(\eta\mu x\right)' dx = \dfrac{\eta\mu^6 x}{6} + c$
4	$x = t^2 + 5 \quad \Rightarrow \dfrac{dx}{dt} = 2t$ $y = 2t^3 + 3 \quad \Rightarrow \dfrac{dy}{dt} = 6t^2$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{6t^2}{2t} = 3t$

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α) $x^2+y^2-6x+4y-12=0$

$g=-3\ ,\ \ f=2\ ,\ \ c=-12$

$K(-g\ ,-f)=K(3,-2)$

$R=\sqrt{g^2+f^2-c}\ =\ \sqrt{9+4+12}=\sqrt{25}=5$

β) $0+2^2-0+8-12=0\ \Rightarrow\ A(0,2)$ πάνω στον κύκλο.

γ) $x^2+y^2-6x+4y-12=0$

$$2x+2y\frac{dy}{dx}-6+4\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{3-x}{y+2}$$

$$\lambda_{\varepsilon\varphi}=\left.\frac{3-x}{y+2}\right|_{(0,2)}=\frac{3}{4}$$

εξ. εφ.: $y-y_1=\lambda_{\varepsilon\varphi}(x-x_1)$

$$y-2=\frac{3}{4}(x-0)\ \Rightarrow\ 3x-4y+8=0$$

6

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$

α) $A(5,0)\ ,\ A'(-5,0)$

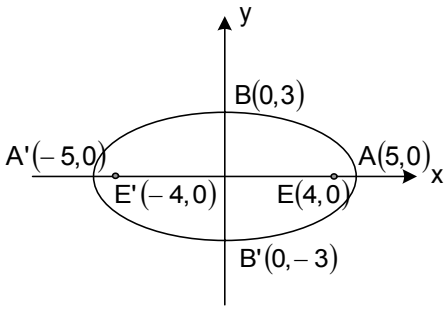
$B(0,3)\ ,\ B'(0,-3)$

$\alpha>\beta\ \Rightarrow\ \gamma^2=\alpha^2-\beta^2$

$\gamma^2=25-9=16\ \Rightarrow\ \gamma=4$

$\Rightarrow\ E(4,0)\ ,E'(-4,0)$

β)



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Ανάλυση σε άθροισμα απλών κλασμάτων:

$$\frac{1}{(x+1)(2x+1)} \equiv \frac{A}{x+1} + \frac{B}{2x+1}$$

$$1 \equiv A(2x+1) + B(x+1)$$

για $x = -1 \;\Rightarrow\; 1 = -A \;\Rightarrow\; A = -1$

για $x = -\frac{1}{2} \;\Rightarrow\; 1 = \frac{1}{2}B \;\Rightarrow\; B = 2$

$$\int \frac{1}{(x+1)(2x+1)}\,dx = \int\left(\frac{-1}{x+1} + \frac{2}{2x+1}\right)dx$$

$$= -\ln|x+1| + \ln|2x+1| + c$$

$$= \ln\left|\frac{2x+1}{x+1}\right| + c$$

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$$A = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} \;,\quad B = \begin{pmatrix} 1 & 1 \\ 4 & -1 \end{pmatrix}$$

α) $$|A| = \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -1 + 2 = 1$$

$$A^{-1} = \frac{1}{1}\begin{pmatrix} -1 & 1 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ -2 & 1 \end{pmatrix}$$

β) $$A \cdot X = B \;\Rightarrow\; X = A^{-1} \cdot B$$

$$X = \begin{pmatrix} -1 & 1 \\ -2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 4 & -1 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -3 \end{pmatrix}$$

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α) Τετραψήφιοι:

Χ	Ε	Δ	Μ
6	5	4	3

$\rightarrow 6\cdot 5\cdot 4\cdot 3 = 360$ τετραψήφιοι

β)Τετραψήφιοι μεγαλύτεροι του 5000:

Χ	Ε	Δ	Μ
2	5	4	3

$\rightarrow 2\cdot 5\cdot 4\cdot 3 = 120$ τετραψήφιοι μεγαλύτεροι του 5000

Α: Τετραψήφιοι μεγαλύτεροι του 5000

$$P(A) = \frac{N(A)}{N(\Omega)} = \frac{120}{360} = \frac{1}{3}$$

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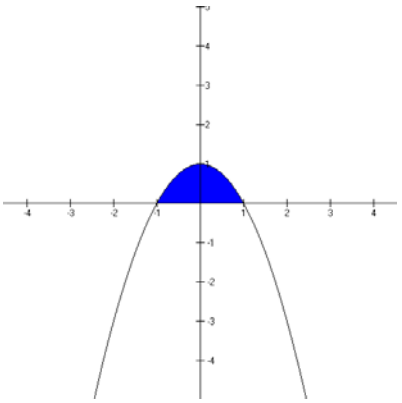


$$\left.\begin{array}{l} y = 1-x^2 \\ y = 0 \end{array}\right\} \Rightarrow 1-x^2 = 0 \Rightarrow x = \pm 1$$

$$V = \pi\int_{-1}^{1}(1-x^2)^2 dx = \pi\int_{-1}^{1}(x^4 - 2x^2 + 1)dx$$

$$= \pi\left[\frac{x^5}{5} - \frac{2x^3}{3} + x\right]_{-1}^{1}$$

$$= \pi\left[\left(\frac{1}{5} - \frac{2}{3} + 1\right) - \left(-\frac{1}{5} + \frac{2}{3} - 1\right)\right]$$

$$= \pi\left(\frac{1}{5} - \frac{2}{3} + 1 + \frac{1}{5} - \frac{2}{3} + 1\right)$$

$$= \frac{16\pi}{15} \text{ κ.μ.}$$

ΜΕΡΟΣ Β

1

$$f(x)=\frac{x^2}{x^2-1}$$

Π.Ο. $\quad x\in R-\{\pm 1\}$

Σημεία Τομής: $\quad x=0 \Rightarrow y=0 \Rightarrow (0,0)$

$\qquad\qquad\quad y=0 \Rightarrow x=0 \Rightarrow (0,0)$

Ακρότατα:
$$\left.\begin{array}{l} y'=\dfrac{2x(x^2-1)-x^2\cdot 2x}{(x^2-1)^2}=\dfrac{2x^3-2x-2x^3}{(x^2-1)^2}=\dfrac{-2x}{(x^2-1)^2} \\ y'=0 \end{array}\right\} \Rightarrow x=0$$

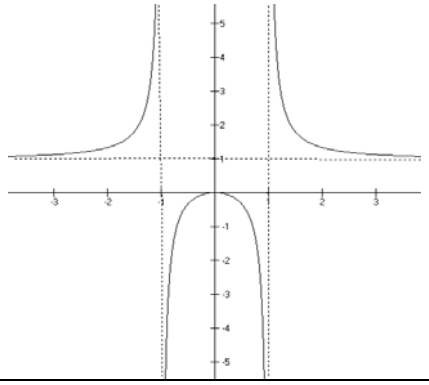
x	$-\infty$		-1		0		1		$+\infty$
y'		+		+	0	−		−	
y		↗		↗	max(0,0)	↘		↘	

Ασύμπτωτες:

$$\left.\begin{array}{l} \lim\limits_{x\to 1^+}\dfrac{x^2}{x^2-1}=\dfrac{1}{0^+}=+\infty \\ \lim\limits_{x\to 1^-}\dfrac{x^2}{x^2-1}=\dfrac{1}{0^-}=-\infty \end{array}\right\} \Rightarrow x=1 \quad \text{K.A.}$$

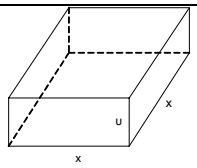
$$\left.\begin{array}{l} \lim\limits_{x\to -1^+}\dfrac{x^2}{x^2-1}=\dfrac{1}{0^-}=-\infty \\ \lim\limits_{x\to -1^-}\dfrac{x^2}{x^2-1}=\dfrac{1}{0^+}=+\infty \end{array}\right\} \Rightarrow x=-1 \quad \text{K.A.}$$

$$\left.\begin{array}{l} \lim\limits_{x\to\pm\infty}\dfrac{x^2}{x^2-1}=\dfrac{\infty}{\infty}\ (\text{απροσδ.}) \\ \lim\limits_{x\to\pm\infty}\dfrac{2x}{2x}=1 \end{array}\right\} \Rightarrow y=1 \text{ O.A.}$$



2	$\int 2x\ln x\,dx$ θέτω: $u=\ln x \Rightarrow du=\frac{1}{x}dx$ $dv=2x\,dx \Rightarrow v=\int 2x\,dx=x^2$ $\int u\,dv=uv-\int v\,du$ $\int 2x\ln x\,dx=x^2\ln x-\int x^2\frac{1}{x}dx$ $=x^2\ln x-\frac{x^2}{2}+c$
3	α) $\binom{11}{3}=\frac{11!}{8!3!}=165$ τρόποι β) $P(A)=\frac{\binom{8}{3}}{\binom{11}{3}}=\frac{\frac{8!}{5!3!}}{165}=\frac{56}{165}$ $P(B)=\frac{\binom{3}{1}\cdot\binom{8}{2}}{\binom{11}{3}}=\frac{3\cdot\frac{8!}{6!2!}}{165}=\frac{84}{165}$ $P(\Gamma)=\frac{\binom{3}{3}}{\binom{11}{3}}=\frac{1}{165}$

4



α) $V = \alpha \cdot \beta \cdot \gamma \;\Rightarrow\; V = x^2 \cdot \upsilon \;\Rightarrow\; 27 = x^2 \cdot \upsilon \;\Rightarrow \upsilon = \dfrac{27}{x^2}$

β) $E_{o\lambda} = 2(\alpha\beta + \beta\gamma + \alpha\gamma) \;\Rightarrow\; E_{o\lambda} = 2\left(x^2 + x\upsilon + x\upsilon\right)$

$$E(x) = 2x^2 + 4x\frac{27}{x^2} = 2x^2 + \frac{108}{x}$$

γ) $$\left.\begin{array}{l} \dfrac{dE}{dx} = 4x - \dfrac{108}{x^2} \\ \dfrac{dE}{dx} = 0 \end{array}\right\} \Rightarrow \frac{4x^3 - 108}{x^2} = 0 \;\Rightarrow\; x^3 = 27 \;\Rightarrow\; x = 3\,\text{cm}$$

$$\frac{d^2E}{dx^2} = 4 + \frac{216}{x^3}\bigg|_{x=3} = 12 > 0 \;\Rightarrow\; \min$$

$$E(3) = 2 \cdot 9 + \frac{108}{3} = 54\,\text{cm}^2$$

5

α) $xy = 9$, $A\left(3t, \frac{3}{t}\right)$

$y + xy' = 0 \Rightarrow y' = -\frac{y}{x}$

$\lambda_{\varepsilon\varphi} = -\frac{y}{x}\Big|_{\left(3t, \frac{3}{t}\right)} = -\frac{3}{t \cdot 3t} = -\frac{1}{t^2}$

εξ. εφαπτ.: $y - y_1 = \lambda_{\varepsilon\varphi}(x - x_1)$

$y - \frac{3}{t} = -\frac{1}{t^2}(x - 3t)$

$t^2y - 3t = -x + 3t \Rightarrow x + t^2y = 6t$

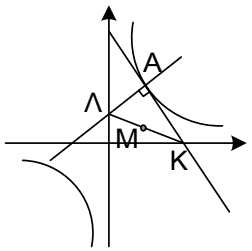
β) $\lambda_{\kappa} = t^2$ εξ. καθ.: $y - y_1 = \lambda_{\kappa}(x - x_1)$

$y - \frac{3}{t} = t^2(x - 3t)$

$ty - 3 = t^3x - 3t^4 \Rightarrow t^3x - ty = 3t^4 - 3$

γ)

$\left.\begin{array}{l} x + t^2y = 6t \\ y = 0 \end{array}\right\} \Rightarrow x = 6t \Rightarrow K(6t, 0)$



$\left.\begin{array}{l} t^3x - ty = 3t^4 - 3 \\ x = 0 \end{array}\right\} \Rightarrow y = \frac{3 - 3t^4}{t} \Rightarrow \Lambda\left(0, \frac{3 - 3t^4}{t}\right)$

Μ μέσο του ΚΛ:

$\left.\begin{array}{l} x_M = \frac{6t + 0}{2} = 3t \\ y_M = \frac{0 + \frac{3 - 3t^4}{t}}{2} = \frac{3 - 3t^4}{2t} \end{array}\right\} \Rightarrow M\left(3t\,,\ \frac{3 - 3t^4}{2t}\right)$

Απαλοιφή παραμέτρου t:

$\left.\begin{array}{l} x = 3t \\ y = \frac{3 - 3t^4}{2t} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t = \frac{x}{3} \\ y = \frac{3 - 3\left(\frac{x}{3}\right)^4}{\frac{2x}{3}} \end{array}\right\} \Rightarrow y = \frac{\frac{81 - x^4}{27}}{\frac{2x}{3}}$

$\Rightarrow y = \frac{81 - x^4}{18x}$ γ.τ. του Μ