

**ΥΠΟΥΡΓΕΙΟ ΠΑΙΔΕΙΑΣ ΚΑΙ ΠΟΛΙΤΙΣΜΟΥ
ΔΙΕΥΘΥΝΣΗ ΑΝΩΤΕΡΗΣ ΚΑΙ ΑΝΩΤΑΤΗΣ ΕΚΠΑΙΔΕΥΣΗΣ
ΥΠΗΡΕΣΙΑ ΕΞΕΤΑΣΕΩΝ**

ΕΞΕΤΑΣΕΙΣ ΓΙΑ ΤΑ ΤΕΧΝΟΛΟΓΙΚΑ ΕΚΠΑΙΔΕΥΤΙΚΑ ΙΔΡΥΜΑΤΑ (Τ.Ε.Ι.)

Μάθημα: ΜΑΘΗΜΑΤΙΚΑ Τ.Ε.Ι.

22 ΙΟΥΝΙΟΥ 2005

ΠΡΟΤΕΙΝΟΜΕΝΕΣ ΛΥΣΕΙΣ ΔΟΚΙΜΙΟΥ

1	<p>(α) $\eta\mu(3x - 50^\circ) = \sigma\upsilon\nu x$ $\eta\mu(3x - 50^\circ) = \eta\mu(90^\circ - x)$ i) $3x - 50^\circ = 360^\circ\kappa + 90^\circ - x$ $4x = 360^\circ\kappa + 140^\circ$ $x = 90^\circ\kappa + 35^\circ, \kappa \in \mathbb{Z}$ ii) $3x - 50^\circ = 360^\circ\kappa + 180^\circ - 90^\circ + x$ $2x = 360^\circ\kappa + 140^\circ$ $x = 180^\circ\kappa + 70^\circ, \kappa \in \mathbb{Z}$</p> <p>$\kappa = 0 \Rightarrow x = 35^\circ$ και $x = 70^\circ$ $\kappa = 1 \Rightarrow x = 125^\circ$</p> <p>(β) $\frac{\eta\mu 3\alpha - \eta\mu \alpha}{\sigma\upsilon\nu \alpha - \sigma\upsilon\nu 3\alpha} = \frac{2\eta\mu \frac{3\alpha - \alpha}{2} \cdot \sigma\upsilon\nu \frac{3\alpha + \alpha}{2}}{2\eta\mu \frac{3\alpha - \alpha}{2} \cdot \eta\mu \frac{3\alpha + \alpha}{2}} =$ $= \frac{\cancel{2\eta\mu \alpha} \cdot \sigma\upsilon\nu 2\alpha}{\cancel{2\eta\mu \alpha} \cdot \eta\mu 2\alpha} = \sigma\phi 2\alpha$</p>	
2	<p>(α) $2\eta\mu 2\alpha(\sigma\upsilon\nu^2 \alpha - \eta\mu^2 \alpha) = 2\eta\mu 2\alpha \sigma\upsilon\nu 2\alpha$ $= \eta\mu 4\alpha$</p> <p>(β) $\eta\mu 4x = 1 \Rightarrow \eta\mu 4x = \eta\mu 90^\circ$ (i) $4x = 360^\circ\kappa + 90^\circ$ $x = 90^\circ\kappa + 22,5^\circ, \kappa \in \mathbb{Z}$ (ii) $4x = 360^\circ\kappa + 180^\circ - 90^\circ$ $x = 90^\circ\kappa + 22,5^\circ, \kappa \in \mathbb{Z}$</p>	

3	<p>(α) $2\sigma\upsilon\nu^2x - 5\sigma\upsilon\nu x + 2 = 0$ θέτω $\sigma\upsilon\nu x = \omega$</p> $2\omega^2 - 5\omega + 2 = 0$ $\omega_{1,2} = \frac{5 \pm \sqrt{9}}{4} = \begin{cases} \omega_1 = 2 \\ \omega_2 = \frac{1}{2} \end{cases}$ <p>για $\omega = 2 \Rightarrow \sigma\upsilon\nu x = 2$ αδύνατη διότι $-1 \leq \sigma\upsilon\nu x \leq 1$</p> <p>για $\omega = \frac{1}{2} \Rightarrow \sigma\upsilon\nu x = \frac{1}{2} \Rightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu 60^\circ$</p> $x = 360^\circ \kappa \pm 60^\circ, \quad \kappa \in \mathbb{Z}$ <p>(β) $(\sigma\upsilon\nu \alpha + \sigma\upsilon\nu \beta)^2 + (\eta\mu \alpha + \eta\mu \beta)^2 = \sigma\upsilon\nu^2 \alpha + 2\sigma\upsilon\nu \alpha \sigma\upsilon\nu \beta + \sigma\upsilon\nu^2 \beta$ $+ \eta\mu^2 \alpha + 2\eta\mu \alpha \eta\mu \beta + \eta\mu^2 \beta$</p> $= 2 + 2(\sigma\upsilon\nu \alpha \sigma\upsilon\nu \beta + \eta\mu \alpha \eta\mu \beta)$ $= 2 + 2\sigma\upsilon\nu (\alpha - \beta)$ $= 2 + 2 \left[2\sigma\upsilon\nu^2 \left(\frac{\alpha - \beta}{2} \right) - 1 \right]$ $= 4\sigma\upsilon\nu^2 \left(\frac{\alpha - \beta}{2} \right)$	
4	<p>(α) $M_8^\epsilon = \frac{8!}{2!} = 20160$</p> <p>(β) $M_7 = 7! = 5040$</p> <p>(γ) $M_6 = 6! = 720$</p>	
5	<p>(α) $\frac{P(A)}{P(A')} = \frac{5}{7} \Rightarrow \frac{P(A)}{1-P(A)} = \frac{5}{7} \Rightarrow 12P(A) = 5 \Rightarrow P(A) = \frac{5}{12}$</p> <p>(β) $P(A - B) = P(A) - P(A \cap B) \Rightarrow \frac{1}{4} = \frac{5}{12} - P(A \cap B) \Rightarrow P(A \cap B) = \frac{1}{6}$</p> <p>(γ) $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow P(A \cup B) = \frac{5}{12} + \frac{1}{3} - \frac{1}{6}$ $\Rightarrow P(A \cup B) = \frac{7}{12}$</p>	

6

(α)

E	Δ	M
4	4	3

→ 4 · 4 · 3 = 48 τριψήφιοι

(β)

i) Τελειώνουν σε μηδέν

E	Δ	M
4	3	1

→ 4 · 3 · 1 = 12 αριθμοί

ii) Τελειώνουν σε 2 ή 4

E	Δ	M
3	3	2

→ 3 · 3 · 2 = 18 αριθμοί

Σύνολο : 12 + 18 = 30 άρτιοι αριθμοί

7

(α) Συνολικές πωλήσεις: 7000+3000+6000+4000 = £20 000

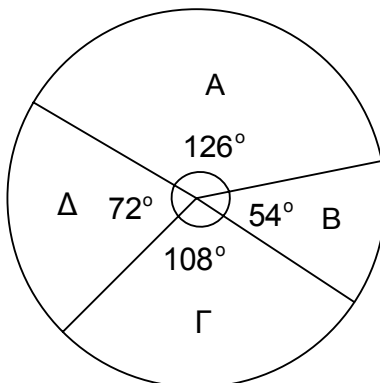
(β)

A : $\frac{7000}{20000} \cdot 360^\circ = 126^\circ$

B : $\frac{3000}{20000} \cdot 360^\circ = 54^\circ$

Γ : $\frac{6000}{20000} \cdot 360^\circ = 108^\circ$

Δ : $\frac{4000}{20000} \cdot 360^\circ = 72^\circ$



8

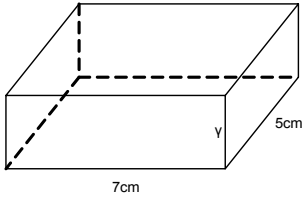
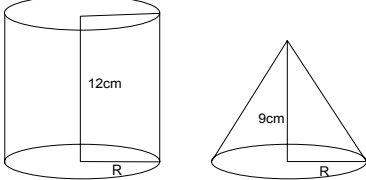
(α) $x_\epsilon = 9$

(β) 7 μέρες

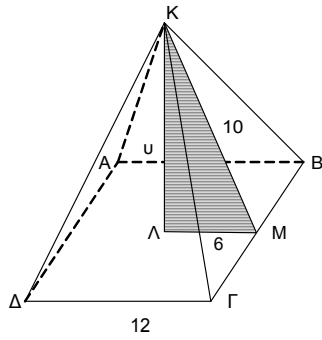
x_i	f_i	$f_i \cdot x_i$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
5	2	10	$(5 - 9)^2 = 16$	$2 \cdot 16 = 32$
6	1	6	$(6 - 9)^2 = 9$	$1 \cdot 9 = 9$
7	2	14	$(7 - 9)^2 = 4$	$2 \cdot 4 = 8$
8	3	24	$(8 - 9)^2 = 1$	$3 \cdot 1 = 3$
9	5	45	$(9 - 9)^2 = 0$	$5 \cdot 0 = 0$
11	3	33	$(11 - 9)^2 = 4$	$3 \cdot 4 = 12$
12	4	48	$(12 - 9)^2 = 9$	$4 \cdot 9 = 36$
	$\Sigma f_i = 20$	$\Sigma f_i \cdot x_i = 180$		$\Sigma f_i \cdot (x_i - \bar{x})^2 = 100$

(γ) $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{180}{20} = 9$

(δ) $\sigma = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}} = \sqrt{\frac{100}{20}} = \sqrt{5} \approx 2,24$

9	<p>(α) $\bar{x} = \frac{\sum x_i}{v_1} \Rightarrow 451 = \frac{\sum x_i}{7} \Rightarrow \sum x_i = 3157 \text{ λίρες}$</p> <p>$\bar{y} = \frac{\sum y_i}{v_2} \Rightarrow 902 = \frac{\sum y_i}{4} \Rightarrow \sum y_i = 3608 \text{ λίρες}$</p> <p>Μέση τιμή όλων:</p> <p>$\bar{z} = \frac{\sum x_i + \sum y_i}{v_3} \Rightarrow \bar{z} = \frac{3157 + 3608}{11} = \frac{6765}{11} = 615 \text{ λίρες}$</p> <p>(β) Νέα μέση τιμή:</p> <p>$\bar{\omega} = \frac{6765 - 805}{10} = \frac{5960}{10} = 596 \text{ λίρες}$</p>	
10	 <p>(α) $E_{\text{ολ}} = 2(\alpha\beta + \alpha\gamma + \beta\gamma) \Rightarrow 166 = 2(5 \cdot 7 + 5 \cdot \gamma + 7 \cdot \gamma)$ $83 = 35 + 12\gamma$ $12\gamma = 48 \Rightarrow \gamma = 4\text{cm}$</p> <p>(β) $V = \alpha \cdot \beta \cdot \gamma$ $= 7 \cdot 5 \cdot 4$ $= 140\text{cm}^3$</p>	
11	 <p>(α) $V_{\text{κυλ}} - V_{\text{κων}} = 324\pi$ $\pi \cdot R^2 \cdot 12 - \frac{\pi \cdot R^2 \cdot 9}{3} = 324\pi$ $9\pi R^2 = 324\pi$ $R^2 = 36 \Rightarrow R = 6\text{cm}$</p> <p>(β) $V_{\text{κυλ}} = \pi R^2 u \Rightarrow V_{\text{κυλ}} = \pi \cdot 36 \cdot 12 \Rightarrow V_{\text{κυλ}} = 432\pi \text{ cm}^3$</p> <p>(γ) $V_{\text{κων}} = \frac{\pi R^2 u}{3} \Rightarrow V_{\text{κων}} = \frac{\pi \cdot 36 \cdot 9}{3} \Rightarrow V_{\text{κων}} = 108\pi \text{ cm}^3$</p>	

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$$(\alpha) \quad E_{\pi} = \frac{\Pi_{\beta} \cdot h}{2}$$

$$240 = \frac{4 \cdot 12 \cdot h}{2}$$

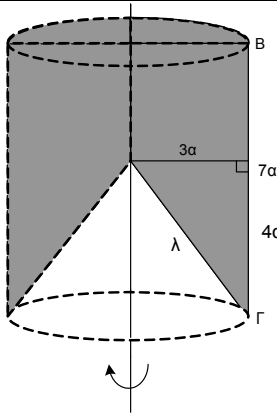
$$h = 10 \text{ cm}$$

$$\begin{aligned}
 (\beta) \quad E_{\text{ολ}} &= E_{\pi} + E_{\beta} \\
 &= 240 + 144 \\
 &= 384 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 (\gamma) \quad \triangle K\Lambda M \text{ ορθογώνιο} &\Rightarrow (KM)^2 = (K\Lambda)^2 + (\Lambda M)^2 \\
 100 &= u^2 + 36 \\
 u &= 8 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 V &= \frac{1}{3} E_{\beta} \cdot u \\
 &= \frac{1}{3} 144 \cdot 8 \\
 &= 384 \text{ cm}^3
 \end{aligned}$$

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$$(\alpha) \quad (\Gamma\Delta)^2 = (3\alpha)^2 + (4\alpha)^2$$

$$\lambda^2 = 9\alpha^2 + 16\alpha^2$$

$$\lambda = 5\alpha$$

$$E_{\text{ολ}} = E_{\text{κυλ}} + E_{\text{κων}} + E_{\beta}$$

$$= 2\pi R u + \pi R \lambda + \pi R^2$$

$$= 2\pi \cdot 3\alpha \cdot 7\alpha + \pi \cdot 3\alpha \cdot 5\alpha + \pi(3\alpha)^2$$

$$= 42\alpha^2 \pi + 15\alpha^2 \pi + 9\alpha^2 \pi$$

$$= 66\alpha^2 \pi$$

$$(\beta) \quad V = V_{\text{κυλ}} - V_{\text{κων}}$$

$$= \pi \cdot R^2 \cdot u_{\text{κυλ}} - \frac{\pi \cdot R^2 \cdot u_{\text{κων}}}{3}$$

$$= \pi \cdot 9\alpha^2 \cdot 7\alpha - \frac{\pi \cdot 9\alpha^2 \cdot 4\alpha}{3}$$

$$= 63\alpha^3 \pi - 12\alpha^3 \pi$$

$$= 51\alpha^3 \pi$$

14

(α)

$$\text{i) } y = x^2 \cdot \eta \mu x$$

$$y' = 2x \cdot \eta \mu x + x^2 \sigma \upsilon \nu x$$

$$\text{ii) } y = \frac{x-2}{x^2+3}$$

$$y' = \frac{1 \cdot (x^2+3) - (x-2) \cdot 2x}{(x^2+3)^2}$$

$$= \frac{x^2+3+4x-2x^2}{(x^2+3)^2}$$

$$= \frac{-x^2+4x+3}{(x^2+3)^2}$$

	(β)	<div style="margin-bottom: 20px;">i) $\int (x^4 + \sqrt{x} + 5)dx = \frac{x^5}{5} + \frac{2\sqrt{x^3}}{3} + 5x + c$</div> <div>ii) $\int (\sigma\upsilon\nu 2x + 2\eta\mu 3x \cdot \sigma\upsilon\nu 4x)dx = \int \sigma\upsilon\nu 2x dx + \int (\eta\mu 7x - \eta\mu x)dx$ $= \int \sigma\upsilon\nu 2x dx + \int \eta\mu 7x dx - \int \eta\mu x dx$ $= \frac{1}{2}\eta\mu 2x - \frac{1}{7}\sigma\upsilon\nu 7x + \sigma\upsilon\nu x + c$</div>						
15	<div style="margin-bottom: 20px;">(α) $y = x^3 - \alpha x + 2$ $x = 0 \Rightarrow y = 2 \Rightarrow (0,2)$ τομή με άξονα Oy. $y' = 3x^2 - \alpha$ $\lambda_{\epsilon\varphi} = 3 \cdot 0 - \alpha \Rightarrow 1 = -\alpha \Rightarrow \alpha = -1$</div> <div style="margin-bottom: 20px;">(β) Εξίσωση εφαπτόμενης στο A : $y - 2 = 1 \cdot (x - 0)$ $x - y + 2 = 0$</div> <div>(γ) Εξίσωση κάθετης στο A : $\lambda_{\epsilon\varphi} = 1 \Rightarrow \lambda_{\kappa\alpha\theta} = -1$ $y - 2 = -1 \cdot (x - 0)$ $x + y - 2 = 0$</div>							
16	<div style="margin-bottom: 20px;">(α) $y = \mu x - x^2$, $\mu > 0$ για $x = 0 \Rightarrow y = 0 \Rightarrow \Sigma.Τομής (0,0)$ για $y = 0 \Rightarrow \mu x - x^2 = 0 \Rightarrow x(\mu - x) = 0$ $\Rightarrow x = 0$ και $x = \mu \Rightarrow \Sigma.Τομής (0,0), (\mu,0)$ Άρα τα σημεία τομής με τους άξονες είναι $(0,0), (\mu,0)$</div> <div style="margin-bottom: 20px;">(β) $\int_0^\mu (\mu x - x^2)dx = \left[\frac{\mu x^2}{2} - \frac{x^3}{3} \right]_0^\mu = \frac{\mu^3}{2} - \frac{\mu^3}{3} = \frac{\mu^3}{6}$ $\frac{\mu^3}{6} = \frac{27}{6} \Rightarrow \mu = 3$</div> <div>(γ) $y' = 3 - 2x$ $y' = 0 \Rightarrow 3 - 2x = 0 \Rightarrow x = \frac{3}{2}$<div style="margin-top: 10px;"><table style="display: inline-table; border-collapse: collapse; text-align: center;"><tr><td style="border-right: 1px solid black; padding: 5px 10px;">x</td><td style="padding: 5px 10px;">$\frac{3}{2}$</td></tr><tr><td style="border-right: 1px solid black; padding: 5px 10px;">y'</td><td style="padding: 5px 10px;">+ 0 -</td></tr><tr><td style="border-right: 1px solid black; padding: 5px 10px;">y</td><td style="padding: 5px 10px;"><div style="display: flex; align-items: center; justify-content: center;"><div style="margin-right: 10px;">\nearrow</div><div style="text-align: center;"><div style="border-top: 1px solid black; width: 20px; margin: 0 auto; position: relative;"><div style="position: absolute; top: -5px; left: 50%; transform: translateX(-50%);">max</div></div><div style="margin-top: 5px;">max</div></div><div style="margin-left: 10px;">\searrow</div></div></td></tr></table></div></div> <div style="margin-top: 20px;">$\text{για } x = \frac{3}{2} \Rightarrow y_{\max} = \frac{9}{4} \Rightarrow \max\left(\frac{3}{2}, \frac{9}{4}\right)$</div>	x	$\frac{3}{2}$	y'	+ 0 -	y	<div style="display: flex; align-items: center; justify-content: center;"><div style="margin-right: 10px;">\nearrow</div><div style="text-align: center;"><div style="border-top: 1px solid black; width: 20px; margin: 0 auto; position: relative;"><div style="position: absolute; top: -5px; left: 50%; transform: translateX(-50%);">max</div></div><div style="margin-top: 5px;">max</div></div><div style="margin-left: 10px;">\searrow</div></div>	
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