

ΥΠΟΥΡΓΕΙΟ ΠΑΙΔΕΙΑΣ ΚΑΙ ΠΟΛΙΤΙΣΜΟΥ
ΔΙΕΥΘΥΝΣΗ ΑΝΩΤΕΡΗΣ ΚΑΙ ΑΝΩΤΑΤΗΣ ΕΚΠΑΙΔΕΥΣΗΣ
ΥΠΗΡΕΣΙΑ ΕΞΕΤΑΣΩΝ

ΕΞΕΤΑΣΕΙΣ ΓΙΑ ΤΑ ΑΝΩΤΕΡΑ ΚΑΙ ΑΝΩΤΑΤΑ ΕΚΠΑΙΔΕΥΤΙΚΑ ΙΔΡΥΜΑΤΑ

ΜΑΘΗΜΑΤΙΚΑ

Τετάρτη 25 Ιουνίου 2003

ΠΡΟΤΕΙΝΟΜΕΝΕΣ ΛΥΣΕΙΣ

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4. $10, 12, x, 18, 13, y, 15, 18, 10, 10$
 $13 = \frac{10+12+x+18+13+y+15+18+10+10}{10}$
 $130 = 3x + 106 \Leftrightarrow 3x = 24 \Leftrightarrow x = 8$
 $\Rightarrow y = 16$

x_i	f_i	$f_i(x_i - \bar{x})^2$
8	1	$(8-13)^2 = 25$
10	3	$3(10-13)^2 = 27$
12	1	$(12-13)^2 = 1$
13	1	$(13-13)^2 = 0$
15	1	$(15-13)^2 = 4$
16	1	$(16-13)^2 = 9$
18	2	$2(18-13)^2 = 50$
		116

$$s = \sqrt{\frac{116}{10}}$$

$$s = \sqrt{11,6}$$

$$s \approx 3,41$$

5. $A = \begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix}$

a) $A^2 = \begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} -3 & 8 \\ -8 & 5 \end{pmatrix}$

$$A^2 - 4A + 7I = \begin{pmatrix} -3 & 8 \\ -8 & 5 \end{pmatrix} - 4\begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix} + 7\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = (0)$$

b) $A^{-1} \cdot A^2 - 4A + 7I = 0 \Rightarrow 7I = 4A - A^2$

$$\Rightarrow 7A^{-1} = 4A^{-1}A - A^{-1}A^2$$

$$7A^{-1} = 4I - A$$

c) $A \cdot X = \begin{pmatrix} 7 & 0 & 14 \\ 0 & -7 & 21 \end{pmatrix}$

$$A^{-1} \cdot A \cdot X = A^{-1} \cdot \begin{pmatrix} 7 & 0 & 14 \\ 0 & -7 & 21 \end{pmatrix}$$

$$X = A^{-1} \cdot \begin{pmatrix} 7 & 0 & 14 \\ 0 & -7 & 21 \end{pmatrix}$$

$$A^{-1} = \frac{1}{7}(4I - A) = \frac{1}{7} \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} - \frac{1}{7} \begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 3 & -2 \\ 2 & 1 \end{pmatrix} \Rightarrow X = \frac{1}{7} \begin{pmatrix} 3 & -2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 7 & 0 & 14 \\ 0 & -7 & 21 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 0 \\ 2 & -1 & 7 \end{pmatrix}$$

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$$\begin{array}{ll}
 6. \quad P(A) = \frac{75}{100} & P(A \cap B) = ; \\
 P(A-B) = \frac{65}{100} & P(B-A) = ; \\
 P(A' \cap B') = \frac{20}{100} & P(A/B) = ;
 \end{array}$$

$$(i) \quad P(A-B) = P(A) - P(A \cap B) \Rightarrow$$

$$\Rightarrow P(A \cap B) = P(A) - P(A-B)$$

$$\Rightarrow P(A \cap B) = \frac{75}{100} - \frac{65}{100} = \boxed{\frac{10}{100}}$$

$$(ii) \quad P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$$

$$\Rightarrow \frac{20}{100} = 1 - P(A \cup B)$$

$$\Rightarrow P(A \cup B) = 1 - \frac{20}{100} = \frac{80}{100}$$

$$\text{Also: } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{80}{100} = \frac{75}{100} + P(B) - \frac{10}{100}$$

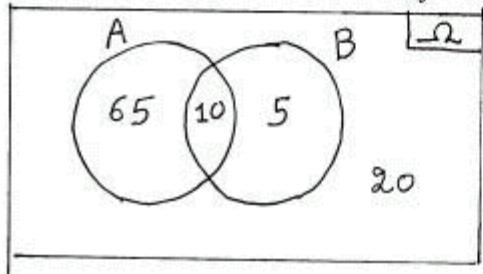
$$\Rightarrow P(B) = \frac{15}{100}$$

$$\Rightarrow P(B-A) = P(B) - P(A \cap B)$$

$$= \frac{15}{100} - \frac{10}{100} = \boxed{\frac{5}{100}}$$

$$(iii) \quad P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{10}{100}}{\frac{15}{100}} = \frac{10}{15} = \boxed{\frac{2}{3}}$$

Zufallsraum: Ω enthält nur jenen ω
Sind A und B Venn.



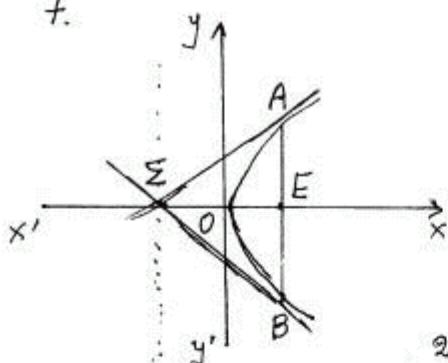
$$(a) \quad P(A \cap B) = \frac{10}{100}$$

$$(b) \quad P(B-A) = \frac{5}{100}$$

$$(c) \quad P(A/B) = \frac{10}{15}$$

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7.



$$\left. \begin{array}{l} y^2 = 4ax, a > 0 \\ AB : x = c \end{array} \right\} \Rightarrow$$

$$\Rightarrow y^2 = 4c^2$$

$$y = \pm 2c$$

$$\Rightarrow A(c, 2c), B(c, -2c)$$

$$y^2 = 4ax \Rightarrow 2y \frac{dy}{dx} = 4a$$

$$\Rightarrow \frac{dy}{dx} = \frac{2a}{y} \Rightarrow \operatorname{tg} \epsilon \varphi_A = \frac{2a}{2a} = 1$$

$$\operatorname{tg} \epsilon \varphi_B = \frac{2a}{-2a} = -1$$

$$\epsilon \varphi_A : y - 2a = (x + a) \Leftrightarrow y = x + 3a \quad \left. \right\} \Rightarrow$$

$$\epsilon \varphi_B : y + 2a = -(x + a) \Leftrightarrow y = -x - 3a \quad \left. \right\} \Rightarrow$$

$$\text{Infinito } \Sigma : \left. \begin{array}{l} y = x + 3a \\ y = -x - 3a \end{array} \right\} \Rightarrow \boxed{\Sigma(-3a, 0)}$$

$$\begin{aligned} V &= \pi \int_{-\alpha}^{\alpha} (x + 3a)^2 dx - \pi \int_{-\alpha}^{\alpha} 4ax dx = \\ &\leq \pi \left[\frac{(x + 3a)^3}{3} \right]_{-\alpha}^{\alpha} - \pi \cdot 2ax^2 \Big|_0^\alpha = \frac{2\pi\alpha^3}{3} \end{aligned}$$

$$\left. \begin{array}{l} V = \frac{2\pi\alpha^3}{3} \\ V = 18\pi \text{ cm}^3 \end{array} \right\} \Rightarrow \frac{2\pi\alpha^3}{3} = 18\pi \Rightarrow$$

$$\Rightarrow \alpha^3 = 27$$

$$\boxed{\alpha = 3}$$

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$$\begin{aligned}
 10. \quad \alpha) \quad M_v - M_{v-1} &= v! - (v-1)! \\
 &\approx v \cdot (v-1)! - (v-1)! \\
 &= (v-1)(v-1)! \\
 &= (v-1) M_{v-1}
 \end{aligned}$$

$$b) \quad 2 + 2M_2 + 3M_3 + \dots + (v-1)M_{v-1} =$$

$$= 2 + M_3 - M_2 + M_4 - M_3 + M_5 - M_4 + \dots + M_v - M_{v-1}$$

$$= 2 - M_2 + M_v$$

$$= 2 - 2! + M_v = M_v$$

MEPOΣ B

$$1. \quad y = \frac{4x}{x^2+4} \quad x \in \mathbb{R}$$

$$\text{für } x=0 \Rightarrow y=0 \quad \text{entfernt } (0,0)$$

$$y' = \frac{4(x^2+4) - 8x^2}{(x^2+4)^2} = \frac{16 - 4x^2}{(x^2+4)^2}$$

$$y' = 0 \Rightarrow x=2, x=-2$$

x	$-\infty$	-2	+2	$+\infty$
y'	-	0	0	-
y	\searrow Min \nearrow Max \searrow	(-2, -1)	(2, 1)	\nearrow

$$\lim_{x \rightarrow +\infty} \frac{4x}{x^2+4} = \left(\frac{+\infty}{+\infty} \right) = \lim_{x \rightarrow +\infty} \frac{4}{2x} = 0^+ \quad \boxed{y=0} \quad \text{asymptote}$$

$$\lim_{x \rightarrow -\infty} \frac{4x}{x^2+4} = \left(\frac{-\infty}{-\infty} \right) = \lim_{x \rightarrow -\infty} \frac{4}{2x} = 0^-$$

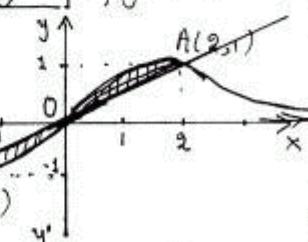
$$\lambda_{AB} = \frac{1+1}{2+2} = \frac{1}{2}$$

$$\text{Eq. AB: } y-1 = \frac{1}{2}(x-2)$$

$$\boxed{2y=x} \Rightarrow \text{Tangente } O(0,0)$$

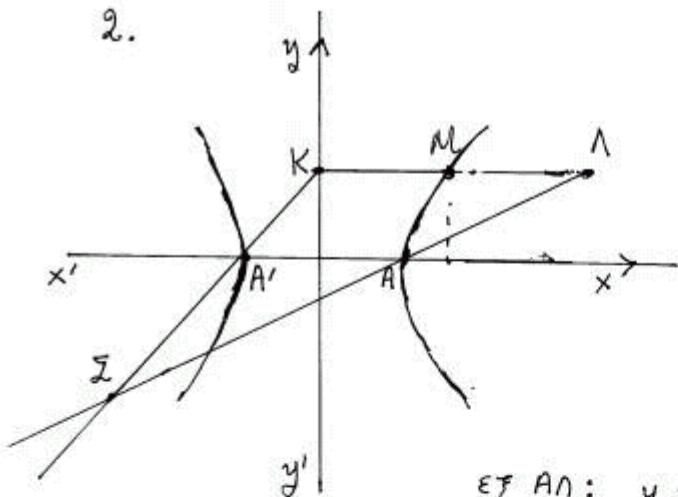
$$E = 2 \int_0^2 \left(\frac{4x}{x^2+4} - \frac{x}{2} \right) dx = 2 \left(2 \ln(x^2+4) - \frac{x^2}{4} \right) \Big|_0^2 =$$

$$= 2(2 \ln 8 - 1 - 2 \ln 4) = 12 \ln 2 - 2 - 8 \ln 2 = \\ = (4 \ln 2 - 2) \text{ T. m}$$



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2.



$$x^2 - y^2 = 1$$

$$M(\epsilon \varphi \delta, \epsilon \varphi \delta)$$

$$K(0, \epsilon \varphi \delta)$$

$$\Gamma(2\epsilon \varphi \delta, \epsilon \varphi \delta)$$

$$A(1, 0)$$

$$A'(-1, 0)$$

$$\vartheta_{AN} = \frac{\epsilon \varphi \delta}{2\epsilon \varphi \delta - 1}$$

$$\vartheta_{A'K} = \epsilon \varphi \delta$$

$$\text{Eq AN: } y = \frac{\epsilon \varphi \delta}{2\epsilon \varphi \delta - 1}(x - 1)$$

$$\text{Eq A'K: } y = \epsilon \varphi \delta(x + 1)$$

Entfernung:

$$\left. \begin{array}{l} y = \epsilon \varphi \delta(x + 1) \\ y = \frac{\epsilon \varphi \delta}{2\epsilon \varphi \delta - 1}(x - 1) \end{array} \right\} \Rightarrow \begin{aligned} x &= \frac{\epsilon \varphi \delta}{1 - 2\epsilon \varphi \delta} \\ y &= \frac{\epsilon \varphi \delta}{1 - 2\epsilon \varphi \delta} \end{aligned}$$

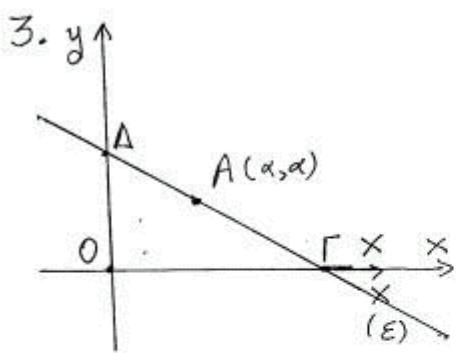
$$\Rightarrow \sqrt{\left(\frac{\epsilon \varphi \delta}{1 - 2\epsilon \varphi \delta} \right)^2 + \left(\frac{\epsilon \varphi \delta}{1 - 2\epsilon \varphi \delta} \right)^2}$$

Erläuterung:

$$\left. \begin{array}{l} x = \frac{\epsilon \varphi \delta}{1 - 2\epsilon \varphi \delta} \\ y = \frac{\epsilon \varphi \delta}{1 - 2\epsilon \varphi \delta} \\ 1 + \epsilon \varphi \delta^2 = 2\epsilon \varphi \delta^2 \end{array} \right\} \Rightarrow \begin{aligned} \epsilon \varphi \delta &= \frac{x}{1+x} \\ \epsilon \varphi \delta y &= y \\ y &= \frac{y}{1+x} \end{aligned}$$

$$1 + \epsilon \varphi \delta^2 = 2\epsilon \varphi \delta^2 \Rightarrow$$

$$\Rightarrow 1 + \frac{y^2}{(1+x)^2} = \frac{x^2}{1+x^2} \Rightarrow \begin{aligned} (1+x)^2 + y^2 &= x^2 \\ 1 + 2x + x^2 + y^2 &= x^2 \\ y^2 &= -2x - 1 \end{aligned}$$



$$(E): y = jx + b$$

$$\text{Für } A(\alpha, \alpha) \Rightarrow$$

$$\alpha = \alpha j + b$$

$$b = \alpha - \alpha j$$

$$\Rightarrow (E): y = jx + \alpha - \alpha j$$

$$\Gamma: y = 0 \Rightarrow x = \frac{\alpha - \alpha j}{j}$$

$$\Gamma\left(\frac{\alpha - \alpha j}{j}, 0\right)$$

$$\Delta: x = 0 \Rightarrow y = \alpha - \alpha j \Rightarrow \Delta(0, \alpha - \alpha j)$$

$$\Rightarrow (0\Gamma) = \frac{\alpha - \alpha j}{j} = \alpha - \frac{\alpha}{j}$$

$$(0\Delta) = \alpha - \alpha j$$

$$\left. \begin{aligned} S &= (0\Gamma) + (0\Delta) \\ S &= \alpha - \frac{\alpha}{j} + \alpha - \alpha j \end{aligned} \right\}$$

$$\Rightarrow S' = \frac{\alpha}{j^2} - \alpha$$

$$S' = 0 \Rightarrow \alpha - \alpha j^2 = 0 \Rightarrow j^2 = 1 \Rightarrow j = \pm 1$$

$$\begin{array}{c|ccc} j & -1 & 0 & +1 \\ \hline S' & - \cancel{\phi} + & \parallel & + \cancel{\phi} - \\ S & \downarrow \min \uparrow & \parallel & \downarrow \end{array}$$

$$S_{\min} \Rightarrow j = -1$$

$$\Rightarrow S_{\min} = \alpha + \alpha + \alpha + \alpha$$

$$\boxed{S_{\min} = 4\alpha}$$