<u>ΠΡΟΤΕΙΝΟΜΕΝΕΣ ΛΥΣΕΙΣ</u>

ΜΕΡΟΣ Α΄

1. $$L=\lim_{x\to 0}\frac{e^{3x}-\sigma\upsilon\nu x-3x}{x^2-\eta\mu 2x+2x}\quad \left(\frac{0}{0}\text{ απροσδιοριστία }\right)\Rightarrow$$

$$L=\lim_{x\to 0}\frac{\left(e^{3x}-\sigma\upsilon\nu x-3x\right)'}{\left(x^2-\eta\mu 2x+2x\right)'}=\lim_{x\to 0}\frac{3e^{3x}-\eta\mu x-3}{2x-2\sigma\upsilon\nu 2x+2}\quad \left(\frac{0}{0}\text{ απροσδιοριστία }\right)\Rightarrow$$

$$L=\lim_{x\to 0}\frac{\left(3e^{3x}-\eta\mu x-3\right)'}{\left(2x-2\sigma\upsilon\nu 2x+2\right)'}=\lim_{x\to 0}\frac{9e^{3x}-\sigma\upsilon\nu x-3}{2+4\eta\mu 2x}=\frac{9+1}{2}=5$$

2. $$a_\kappa=\begin{vmatrix}1 & \kappa & 0\\ 0 & 1 & 6\\ \kappa & 1 & \kappa\end{vmatrix}=\kappa-6-\kappa\left(-6\kappa\right)=6\kappa^2+\kappa-6.$$

$$\sum a_\kappa=\sum_{\kappa=1}^{\nu}\left(6\kappa^2+\kappa-6\right)=6\sum_{\kappa=1}^{\nu}\kappa^2+\sum_{\kappa=1}^{\nu}\kappa-\sum_{\kappa=1}^{\nu}6=\cancel{6}\,\frac{\nu\left(\nu+1\right)\left(2\nu+1\right)}{\cancel{6}}+\frac{\nu\left(\nu+1\right)}{2}-6\nu\ \Rightarrow$$

$$\sum a_\kappa=\frac{\nu\left[\left(2\nu+2\right)\left(2\nu+1\right)+\nu+1-12\right]}{2}=\frac{\nu\left(4\nu^2+6\nu+2+\nu+1-12\right)}{2}=\frac{\nu\left(4\nu^2+7\nu-9\right)}{2}$$

3. $$2\,\tau o\xi\eta\mu 2\chi+\tau o\xi\sigma\upsilon\nu\left(2\sqrt{3}\chi\right)=\frac{\pi}{2}\ (1)$$

Θέτω $\tau o\xi\eta\mu 2\chi = a \Rightarrow \eta\mu a = 2x,\ \ -\frac{\pi}{2} \le a \le \frac{\pi}{2}$,

$\tau o\xi\sigma\upsilon\nu\left(2\sqrt{3}\chi\right) = \beta \Rightarrow \sigma\upsilon\nu\beta = 2\sqrt{3}\chi,\ 0 \le \beta \le \pi$

$$\stackrel{(1)}{\Rightarrow}\ 2a + \beta = \frac{\pi}{2} \Rightarrow 2a = \frac{\pi}{2} - \beta \Rightarrow \sigma\upsilon\nu 2\alpha = \sigma\upsilon\nu\left(\frac{\pi}{2} - \beta\right) \Rightarrow 1 - 2\eta\mu^2\alpha = \eta\mu\beta \Rightarrow$$

$$1 - 2\cdot 4\chi^2 = \sqrt{1 - 12\chi^2} \Rightarrow \left(1 - 8x^2\right)^2 = 1 - 12x^2 \Rightarrow 1 - 16x^2 + 64x^4 = 1 - 12x^2 \Rightarrow 64x^4 - 4x^2 = 0$$

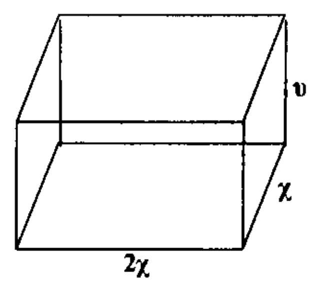
$$\Rightarrow 4x^2\left(4x - 1\right)\left(4x + 1\right) = 0 \Rightarrow x = 0,\ x = \frac{1}{4},\ x = -\frac{1}{4}$$

$x = 0\ :\ 2\tau o\xi\eta\mu 0 + \tau o\xi\sigma\upsilon\nu 0 = 0 + \frac{\pi}{2} = \frac{\pi}{2}$ ✔ $\Rightarrow$ $x = 0$ δεκτή

$x = \frac{1}{4}:\ \ 2\tau o\xi\eta\mu\frac{1}{2} + \tau o\xi\sigma\upsilon\nu\frac{\sqrt{3}}{2} = 2\cdot\frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{2}$ ✔ $\Rightarrow x = \frac{1}{4}$ δεκτή

$x = -\frac{1}{4}:\ 2\tau o\xi\eta\mu\left(-\frac{1}{2}\right) + \tau o\xi\sigma\upsilon\nu\left(-\frac{\sqrt{3}}{2}\right) = \left(-\frac{\pi}{3}\right) + \frac{5\pi}{6} = \frac{\pi}{2}$ ✔ $\Rightarrow x = -\frac{1}{4}$ δεκτή

4. (α) $2x^2\cdot\upsilon = 10 \Rightarrow \upsilon = \frac{10}{2x^2} \Rightarrow \upsilon = \frac{5}{x^2}$.



$$K = 2x^2 + 6x\cdot\upsilon\cdot\frac{16}{15} \Rightarrow K = 2x^2 + 6x\cdot\frac{5}{x^2}\cdot\frac{16}{15} \Rightarrow K = 2x^2 + \frac{32}{x^2}$$

(β) $\frac{dK}{dx} = 4x - \frac{32}{x^2} \Rightarrow \frac{dK}{dx} = \frac{4\left(x^3 - 8\right)}{x^2},\ \ \frac{dK}{dx} = 0 \Rightarrow$

$x^3 - 8 = 0 \Rightarrow x = 2$

χ		2	
$\frac{dK}{dx}$	−	0	+
K	↘	min	↗

Άρα για να είναι ελάχιστο το κόστος πρέπει οι διαστάσεις της δεξαμενής να είναι 2m και 4m.

(γ) Για $x = 2 \Rightarrow K_{\min} = 2\cdot 2^2 + \frac{32}{2} \Rightarrow K_{\min} = 8 + 16 \Rightarrow K_{\min} = £24$

5. $\left(3 + 2x\right)^\nu,\ \nu \in \mathbb{N}$, $\qquad T_{\kappa+1} = \binom{\nu}{\kappa} 3^{\nu-\kappa} 2^\kappa \chi^\kappa$

(α) $$\frac{\binom{\nu}{2}3^{\nu-2}2^2}{\binom{\nu}{3}3^{\nu-3}2^3} = \frac{3}{4} \Rightarrow \frac{\frac{\nu\left(\nu-1\right)}{1\cdot 2}\cdot 3}{\frac{\nu\left(\nu-1\right)\left(\nu-2\right)}{1\cdot 2\cdot 3}\cdot 2} = \frac{3}{4} \Rightarrow \frac{9}{2\left(\nu-2\right)} = \frac{3}{4} \Rightarrow \nu - 2 = 6 \Rightarrow \boxed{\nu = 8}$$

(β) Για κ=5 $T_6 = \binom{8}{5} 3^{8-5} 2^5 \chi^5 \Rightarrow T_6 = 48384\, x^5$. Άρα ο συντελεστής είναι 48384

6. Θεωρούμε τα ενδεχόμενα:

Γ: γνωρίζει την σωστή απάντηση, Τ: απαντά στην τύχη, Σ: απαντά σωστά

$$\mathrm{P}(\Gamma) = \frac{3}{5},\quad \mathrm{P}(\mathrm{T}) = \frac{2}{5},\quad \mathrm{P}(\Sigma/\Gamma) = 1,\quad \mathrm{P}(\Sigma/\mathrm{T}) = \frac{1}{4},\quad \mathrm{P}(\Sigma'/\mathrm{T}) = \frac{3}{4}$$

$$P(\Sigma) = \mathrm{P}(\Gamma \cap \Sigma) + \mathrm{P}(\mathrm{T} \cap \Sigma) = \mathrm{P}(\Gamma)\cdot\mathrm{P}(\Sigma/\Gamma) + \mathrm{P}(\mathrm{T})\cdot\mathrm{P}(\Sigma/\mathrm{T}) = \frac{3}{5}\cdot 1 + \frac{2}{5}\cdot\frac{1}{4} = \frac{3}{5} + \frac{1}{10} = \frac{7}{10}$$

$$\mathrm{P}(\Gamma/\Sigma) = \frac{\mathrm{P}(\Gamma \cap \Sigma)}{\mathrm{P}(\Sigma)} = \frac{\frac{3}{5}}{\frac{7}{10}} = \frac{6}{7}$$

7. α) $M = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 3x \\ 2y \end{pmatrix} \Rightarrow \begin{matrix} X = 3x \Rightarrow x = \dfrac{X}{3} \\ Y = 2y \Rightarrow y = \dfrac{Y}{2} \end{matrix} \Rightarrow$

$\dfrac{X^2}{9} - \dfrac{Y^2}{4} = 1$ που είναι η υπερβολή (κ)

(β) $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1,\ \begin{matrix} a = 3 \\ \beta = 2 \end{matrix} \Rightarrow \gamma^2 = \alpha^2 + \beta^2 \Rightarrow \gamma^2 = 9 + 4 \Rightarrow \gamma^2 = 13 \Rightarrow \gamma = \sqrt{13}$

Κορυφές: $A(3,0)$, $A'(-3,0)$. Εστίες: $E\left(\sqrt{13},0\right)$, $E'\left(-\sqrt{13},0\right)$.

Εκκεντρότητα: $\varepsilon = \dfrac{\gamma}{\alpha} \Rightarrow \varepsilon = \dfrac{\sqrt{13}}{3}$. Ασύμπτωτες: $y = \pm\dfrac{2}{3}x$

8. (α) $\dfrac{d}{dx}\left(\dfrac{\alpha\eta\mu\chi}{\beta + \alpha\sigma\upsilon\nu\chi}\right) = \dfrac{(\beta + \alpha\sigma\upsilon\nu\chi)\alpha\sigma\upsilon\nu\chi - \alpha\eta\mu\chi(-\alpha\eta\mu\chi)}{(\beta + \alpha\sigma\upsilon\nu\chi)^2} =$

$$= \frac{\alpha\beta\sigma\upsilon\nu\chi + \alpha^2\sigma\upsilon\nu^2\chi + \alpha^2\eta\mu^2\chi}{(\beta + \alpha\sigma\upsilon\nu\chi)^2} = \frac{\alpha\beta\sigma\upsilon\nu\chi + \alpha^2}{(\beta + \alpha\sigma\upsilon\nu\chi)^2} = \frac{\alpha\beta\sigma\upsilon\nu\chi + \beta^2 + \alpha^2 - \beta^2}{(\beta + \alpha\sigma\upsilon\nu\chi)^2} =$$

$$= \frac{\beta\,\cancel{(\alpha\sigma\upsilon\nu\chi + \beta)}}{(\beta + \alpha\sigma\upsilon\nu\chi)^{\cancel{2}}} + \frac{\alpha^2 - \beta^2}{(\beta + \alpha\sigma\upsilon\nu\chi)^2} = \frac{\beta}{\beta + \alpha\sigma\upsilon\nu\chi} + \frac{\alpha^2 - \beta^2}{(\beta + \alpha\sigma\upsilon\nu\chi)^2}$$

(β) $\int_0^{\frac{\pi}{2}} \frac{dx}{5+4\sigma\upsilon\nu\chi}$ Θέτω $t = \varepsilon\varphi\frac{x}{2} \Rightarrow \frac{x}{2} = \tau o\xi\varepsilon\varphi t \Rightarrow dx = \frac{2dt}{1+t^2}$, $\begin{array}{c|c|c} x & 0 & \frac{\pi}{2} \\ \hline t & 0 & 1 \end{array}$

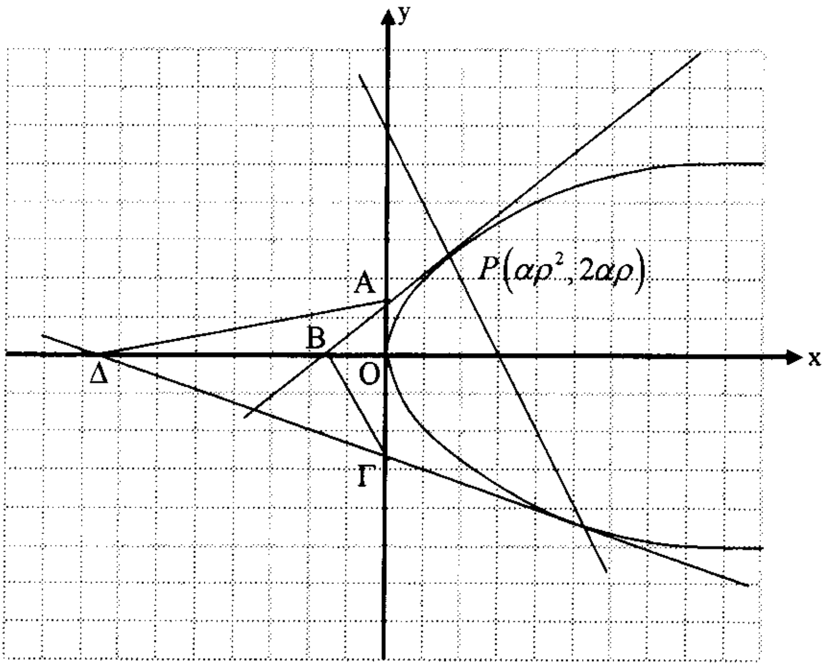
$$\int_0^{\frac{\pi}{2}} \frac{dx}{5+4\sigma\upsilon\nu\chi} = \int_0^1 \frac{\frac{2dt}{1+t^2}}{5+4\frac{1-t^2}{1+t^2}} = \int_0^1 \frac{2dt}{5\left(1+t^2\right)+4\left(1-t^2\right)} = \int_0^1 \frac{2dt}{9+t^2} = \frac{2}{3}\left[\tau o\xi\varepsilon\varphi\left(\frac{t}{3}\right)\right]_0^1 =$$

$$= \frac{2}{3}\left(\tau o\xi\varepsilon\varphi\frac{1}{3} - \tau o\xi\varepsilon\varphi 0\right) = \frac{2}{3}\left(\tau o\xi\varepsilon\varphi\frac{1}{3} - 0\right) = \frac{2}{3}\tau o\xi\varepsilon\varphi\frac{1}{3}$$

(γ) Θέτοντας α = 4, β = 5 στη σχέση του ερωτήματος (α) και ολοκληρώνοντας τα δύο μέλη παίρνουμε: $\left[\frac{4\eta\mu\chi}{5+4\sigma\upsilon\nu\chi}\right]_0^{\frac{\pi}{2}} = \int_0^{\frac{\pi}{2}} \frac{5dx}{5+4\sigma\upsilon\nu\chi} - \int_0^{\frac{\pi}{2}} \frac{9dx}{\left(5+4\sigma\upsilon\nu\chi\right)^2} \Rightarrow$

$\frac{4}{5} = \frac{10}{3}\tau o\xi\varepsilon\varphi\frac{1}{3} - 9\int_0^{\frac{\pi}{2}} \frac{dx}{\left(5+4\sigma\upsilon\nu\chi\right)^2} \Rightarrow \int_0^{\frac{\pi}{2}} \frac{dx}{\left(5+4\sigma\upsilon\nu\chi\right)^2} = \frac{10}{27}\tau o\xi\varepsilon\varphi\frac{1}{3} - \frac{4}{25}$.

9. (α) $y^2 = 4ax \Rightarrow 2y\frac{dy}{dx} = 4a \Rightarrow \frac{dy}{dx} = \frac{2a}{y}$, $P\left(\alpha\rho^2 2\alpha\rho\right) \Rightarrow \lambda_{\varepsilon\varphi} = \frac{2\alpha}{2\alpha\rho} = \frac{1}{\rho}$



Εξίσωση εφαπτομένης:

$y - 2a\rho = \frac{1}{\rho}\left(\chi - \alpha\rho^2\right) \Rightarrow$

$\rho y - 2a\rho^2 = x - a\rho^2 \Rightarrow$

$\boxed{x - \rho y + a\rho^2 = 0}$

$\lambda_{\varepsilon\varphi} = \frac{1}{\rho} \Rightarrow \lambda_{\kappa\alpha\theta} = -\rho$

Εξίσωση κάθετης:

$y - 2a\rho = -\rho\left(\chi - a\rho^2\right) \Rightarrow$

$\boxed{y + \rho x = 2a\rho + a\rho^3}$

$\lambda_{TP} = \frac{y_P - y_T}{x_P - x_T} \Rightarrow$

$\lambda_{TP} = \frac{2a\rho - 2at}{a\rho^2 - at^2} \Rightarrow \lambda_{TP} = \frac{2\cancel{(\rho - t)}}{\cancel{(\rho - t)}(\rho + t)}$, $\rho \neq t \Rightarrow \lambda_{TP} = \frac{2}{(\rho + t)}$, $\lambda_{\kappa\alpha\theta} = -\rho \Rightarrow$

$\frac{2}{(\rho + t)} = -\rho \Rightarrow -\rho^2 - \rho t - 2 = 0 \Rightarrow \boxed{\rho^2 + \rho t + 2 = 0}$

(γ) (i) Εφαπτομένη στο σημείο $P\left(\alpha\rho^2, 2\alpha\rho\right)$: $x-\rho y+a\rho^2=0$

για χ=0 $\Rightarrow \rho y = a\rho^2 \Rightarrow y = a\rho \Rightarrow A\left(0,a\rho\right)$, για y=0 $\Rightarrow x = a\rho^2 \Rightarrow B\left(-a\rho^2,0\right)$

Εφαπτομένη στο σημείο $T\left(at^2,2at\right)$: $x-ty+at^2=0$

για χ=0 $\Rightarrow y=at \Rightarrow \Gamma\left(0,at\right)$, για y=0 $\Rightarrow x=at^2 \Rightarrow \Delta\left(-at^2,0\right)$

Από τη σχέση $\rho^2+\rho t+2=0$ έχουμε $t=-\dfrac{2+\rho^2}{\rho} \Rightarrow$

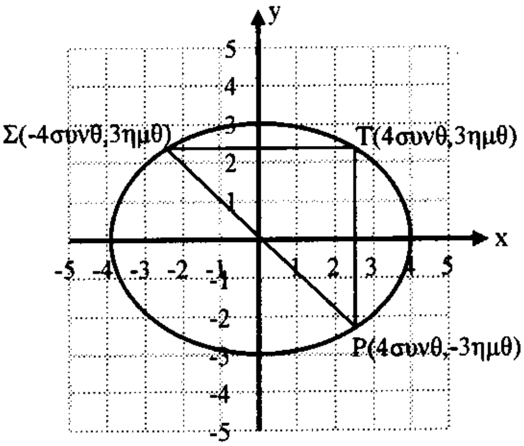
$$\Gamma\left(0,-\frac{a\left(2+\rho^2\right)}{\rho}\right),\ \Delta\left(-\frac{a\left(2+\rho^2\right)^2}{\rho},0\right)$$

$$E_{O A\Delta}=\frac{1}{2}\left|y_A\right|\cdot\left|x_\Delta\right|=\frac{1}{2}\cdot\frac{a\left(2+\rho^2\right)^2}{\rho}\cdot a\rho=\frac{a^2\left(2+\rho^2\right)^2}{2\rho},$$

$$E_{OB\Gamma}=\frac{1}{2}\left|y_\Gamma\right|\cdot\left|x_B\right|=\frac{1}{2}\cdot\frac{a\left(2+\rho^2\right)^2}{\rho}\cdot a\rho^2=\frac{a^2\rho\left(2+\rho^2\right)^2}{2} \Rightarrow$$

$$\frac{E_{OA\Delta}}{E_{OB\Gamma}}=\frac{\dfrac{a^2\left(2+\rho^2\right)^2}{2\rho}}{\dfrac{a^2\rho\left(2+\rho^2\right)}{2}}=\frac{2+\rho^2}{\rho^2}=1+\frac{2}{\rho^2}>1 \Rightarrow E_{OA\Delta}>E_{OB\Gamma}$$

(ii) $\lim\limits_{\rho\to\infty}\dfrac{E_{OA\Delta}}{E_{OB\Gamma}}=\lim\limits_{\rho\to\infty}\dfrac{\rho^2+2}{\rho^2}=\lim\limits_{\rho\to\infty}\left(1+\dfrac{2}{\rho^2}\right)=1$



10. $\dfrac{x^2}{16}+\dfrac{y^2}{9}=1$

(α) Εξίσωση ΡΣ : $\dfrac{x+4\sigma\upsilon\nu\theta}{8\sigma\upsilon\nu\theta}=\dfrac{y-3\eta\mu\theta}{-6\eta\mu\theta} \Rightarrow$

$-6\eta\mu\theta\cdot x-24\eta\mu\theta\sigma\upsilon\nu\theta=8\sigma\upsilon\nu\theta\cdot y-24\eta\mu\theta\sigma\upsilon\nu\theta \Rightarrow$

$6\eta\mu\theta\cdot x+8\sigma\upsilon\nu\theta\cdot y=0 \Rightarrow 3\eta\mu\theta\cdot x+4\sigma\upsilon\nu\theta\cdot y=0$

(β) $\dfrac{2x}{16}+\dfrac{2y}{9}\cdot\dfrac{dy}{dx}=0 \Rightarrow \dfrac{y}{9}\dfrac{dy}{dx}=-\dfrac{x}{16} \Rightarrow \dfrac{dy}{dx}=-\dfrac{9x}{16y}$, $\lambda_{\kappa\alpha\theta}=\dfrac{16y}{9x}=\dfrac{16\cdot 3\eta\mu\theta}{9\cdot 4\sigma\upsilon\nu\theta}=\dfrac{4\eta\mu\theta}{3\sigma\upsilon\nu\theta}$,

$3\eta\mu\theta)$

Εξίσωση κάθετης: $y-3\eta\mu\theta=\dfrac{4\eta\mu\theta}{3\sigma\upsilon\nu\theta}\left(x-4\sigma\upsilon\nu\theta\right) \Rightarrow$

$3\sigma\upsilon\nu\theta\cdot y-9\eta\mu\theta\cdot\sigma\upsilon\nu\theta=4\eta\mu\theta\cdot x-16\eta\mu\theta\cdot\sigma\upsilon\nu\theta \Rightarrow 4\eta\mu\theta\cdot x-3\sigma\upsilon\nu\theta\cdot y=7\eta\mu\theta\cdot\sigma\upsilon\nu\theta$.

(γ) $\left.\begin{aligned} 4\eta\mu\theta\cdot x - 3\sigma\upsilon\nu\theta\cdot y &= 7\eta\mu\theta\cdot\sigma\upsilon\nu\theta \\ 3\eta\mu\theta\cdot x + 4\sigma\upsilon\nu\theta\cdot y &= 0 \end{aligned}\right|\begin{matrix}4\\3\end{matrix} \Rightarrow \begin{aligned} 16\eta\mu\theta\cdot x - 12\sigma\upsilon\nu\theta\cdot y &= 28\eta\mu\theta\cdot\sigma\upsilon\nu\theta \\ 9\eta\mu\theta\cdot x + 12\sigma\upsilon\nu\theta\cdot y &= 0 \end{aligned} \Rightarrow$

$25\eta\mu\theta\cdot x = 28\eta\mu\theta\cdot\sigma\upsilon\nu\theta \Rightarrow \underline{x = \frac{28}{25}\sigma\upsilon\nu\theta}$, $\quad y = -\frac{3\eta\mu\theta\cdot x}{4\sigma\upsilon\nu\theta} \Rightarrow y = -\frac{3\eta\mu\theta}{4\sigma\upsilon\nu\theta}\cdot\frac{28}{25}\sigma\upsilon\nu\theta \Rightarrow$

$\underline{y = -\frac{21}{25}\eta\mu\theta}$

$$\left.\begin{aligned} \sigma\upsilon\nu\theta &= \frac{25}{28}x \\ \eta\mu\theta &= -\frac{25}{28}y \\ \eta\mu^2\theta + \sigma\upsilon\nu^2\theta &= 1 \end{aligned}\right\} \Rightarrow \frac{x^2}{\left(\frac{28}{25}\right)^2} + \frac{y^2}{\left(\frac{21}{25}\right)^2} 1$$ που είναι έλλειψη

<u>ΜΕΡΟΣ Β</u>

1. (α) $y = \dfrac{3x+2}{x(x-2)}$, $x(x-2) \neq 0 \Rightarrow x \neq 0 \wedge y \neq 0$ π.ο. $x \in \mathbb{R} - \{0, 2\}$

χ = 0 δεν βρίσκεται στο πεδίο ορισμού άρα η καμπύλη δεν τέμνει τον άξονα των y.

y = 0 ⇒ $3x + 2 = 0 \Rightarrow x = -\frac{3}{2}$ άρα η καμπύλη τέμνει τον άξονα των χ στο σημείο $\left(-\frac{3}{2}, 0\right)$

$$\frac{dy}{dx} = \frac{x(x-2)3 - (3x+2)(2x-2)}{x^2(x-2)^2} = \frac{3x^2 - 6x - 6x^2 + 6x - 4x + 4}{x^2(x-2)^2} = \frac{-3x^2 - 4x + 4}{x^2(x-2)^2} = \frac{-3(x+2)\left(x - \frac{3}{2}\right)}{x^2(x-2)^2}$$

$$\frac{dy}{dx} = 0 \Rightarrow x = -2,\ x = \frac{2}{3}$$

x	$-\infty$		-2		0		$\frac{2}{3}$		0		$+\infty$
$\frac{dy}{dx}$		$-$	0	$+$	‖	$+$	0	$-$	‖	$-$	
y		↘	min	↗	‖	↗	max	↘	‖	↘	

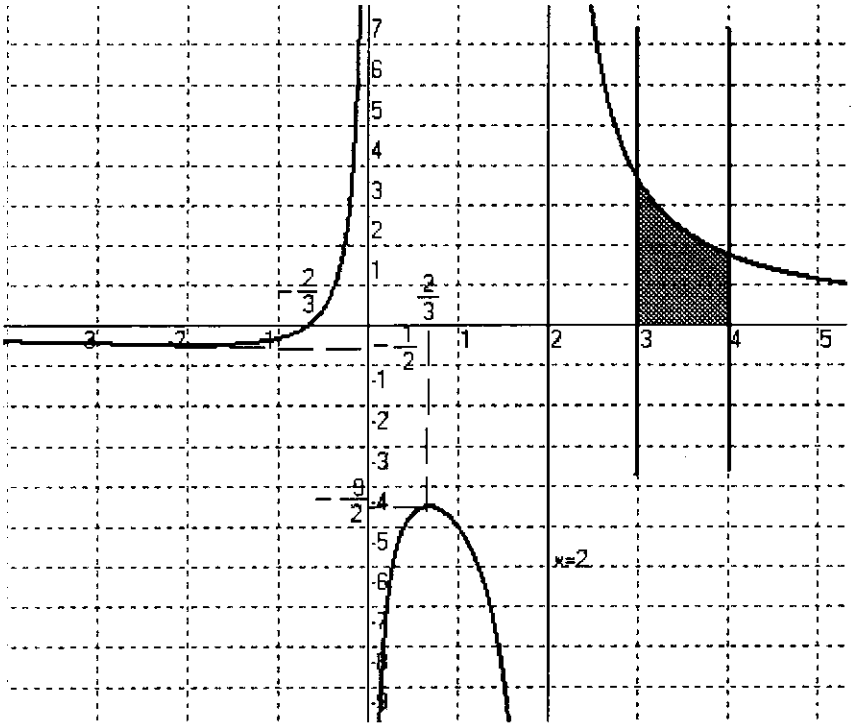
$x = -1 \Rightarrow y_{\min} = -\frac{1}{2}$

$x = \frac{2}{3} \Rightarrow y_{\max} = -\frac{9}{2}$

$\min\left(-2, -\frac{1}{2}\right)$, $\qquad \max\left(\frac{3}{2}, -\frac{9}{2}\right)$

Κατακόρυφες ασύμπτωτες: $x = 0$, $\quad x = 2$

$\lim\limits_{x\to\pm\infty} \dfrac{3x+2}{x^2 - 2x} = \lim\limits_{x\to\pm\infty} \dfrac{\cancel{x}\left(3 + \frac{2}{x}\right)}{x^{\cancel{2}}\left(1 - \frac{2}{x}\right)} = 0$. Άρα η ευθεία $y = 0$ δηλαδή ο άξονας των χ είναι οριζόντια ασύμπτωτη της καμπύλης και προς τα αριστερά και προς τα δεξιά.



(β) $\dfrac{3x+2}{x(x-2)} \equiv \dfrac{A}{x}+\dfrac{B}{x-2}$

$\Rightarrow 3x+2 \equiv A(x-2)+Bx$

για χ=0 $\Rightarrow A=-1$

για χ=2 $\Rightarrow B=4$

Άρα $\dfrac{3x+2}{x(x-2)}=\dfrac{4}{x-2}-\dfrac{1}{x}$

$E=\int_3^4\left(\dfrac{4}{x-2}-\dfrac{1}{x}\right)dx \Rightarrow$

$E=\left[4\ln|x-2|-\ln|x|\right]_3^4$

$E=4\ln 2-\ln 4-(4\ln 1-\ln 3)$

$E=2\ln 4-\ln 4+\ln 3 \Rightarrow$

$E=\ln 4+\ln 3 \Rightarrow E=\ln 12 \Rightarrow a=12$

2. (ε): $(\varepsilon): \vec{r}=(-6\vec{i}+2\vec{j})+\lambda(2\vec{i}-\vec{j}+3\vec{k})$

για λ=0 $\Rightarrow \vec{r}=-6\vec{i}+2\vec{j} \Rightarrow A(-6,2,0)$, για λ=1 $\Rightarrow \vec{r}=-4\vec{i}+\vec{j}+3\vec{k} \Rightarrow B(-4,1,3)$

$(-6\vec{i}+2\vec{j})\cdot(-\vec{i}+\vec{j}+\vec{k})=6+2=8$ άρα $A\in\Pi$

$(-4\vec{i}+\vec{j}+3\vec{k})\cdot(-\vec{i}+\vec{j}+\vec{k})=4+1+3=8$ άρα $B\in\Pi$

(β) $\overrightarrow{ON}=(2\lambda-6)\vec{i}+(-\lambda+2)\vec{j}+3\lambda\vec{k}$, $\overrightarrow{ON}\perp(2\vec{i}-\vec{j}+3\vec{k}) \Rightarrow \overrightarrow{ON}\cdot(2\vec{i}-\vec{j}+3\vec{k})=0 \Rightarrow$

$(2\lambda-6)\cdot 2+(-\lambda+2)\cdot(-1)+3\lambda\cdot 3=0 \Rightarrow 4\lambda-12+\lambda-2+9\lambda=0 \Rightarrow 14\lambda=14 \Rightarrow \lambda=1$

$\overrightarrow{ON}=-4\vec{i}+\vec{j}+3\vec{k} \Rightarrow N(-4,1,3)$

(γ) $\overrightarrow{ON}\times(2\vec{i}-\vec{j}+3\vec{k})=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ -4 & 1 & 3\\ 2 & -1 & 3\end{vmatrix}=6\vec{i}-\vec{j}\cdot(-12-6)+(4-2)\cdot\vec{k}=6\vec{i}+18\vec{j}+2\vec{k}=$

$=2(3\vec{i}+9\vec{j}+\vec{k}) \Rightarrow \vec{n}=3\vec{i}+9\vec{j}+\vec{k}$

$\vec{r}\cdot\vec{n}=0 \Rightarrow \vec{r}\cdot(3\vec{i}+9\vec{j}+\vec{k})=0$

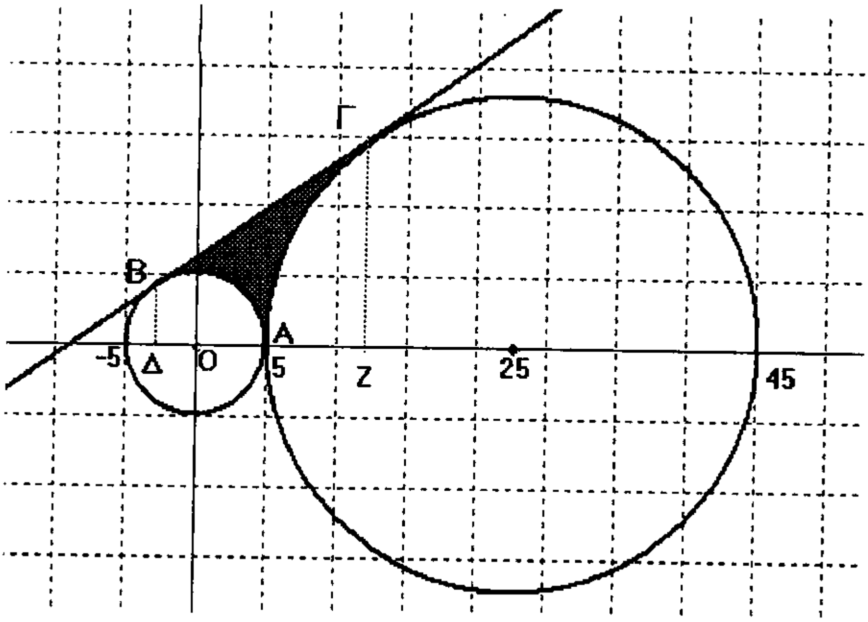
3. (α) $\kappa_1: x^2+y^2-25=0 \Rightarrow K_1(0,0)$, $R_1=5$

$\kappa_2: x^2+y^2-50x+225=0 \Rightarrow K_2(25,0)$, $R_2=\sqrt{625-225}=\sqrt{400}=20$

$|K_1K_2|=R_1+R_2$ άρα οι δύο κύκλοι εφάπτονται εξωτερικά

(β) $\left.\begin{array}{l} x^2+y^2=25 \\ x^2+y^2-50x=-225 \end{array}\right\} \overset{(-)}{\Rightarrow} 50x=250 \Rightarrow x=5 \Rightarrow y=0$ σημείο επαφής Α(5,0)

(γ)



$$\left.\begin{aligned} y &= \frac{25+3x}{4} \\ x^2+y^2 &= 25 \end{aligned}\right\} \Rightarrow$$

$$x^2+\frac{(25+3x)^2}{16}=25 \Rightarrow$$

$$25x^2+150x+225=0$$

$$\Rightarrow x^2+6x+9=0 \Rightarrow$$

$$(x+3)^2=0 \Rightarrow x=-3,$$

$$y=4 \Rightarrow B(-3,4)$$

$$\left.\begin{aligned} y &= \frac{25+3x}{4} \\ x^2+y^2-50x+225 &= 0 \end{aligned}\right\} \Rightarrow x^2+\frac{(25+3x)^2}{16}-50x+225=0 \Rightarrow 25x^2-650x+4225=0 \Rightarrow$$

$$x^2-26x+169=0 \Rightarrow (x-13)^2=0 \Rightarrow x=13,\ y-16 \Rightarrow \Gamma(13,16)$$

$$V=V_{\kappa o\lambda.\kappa\omega\nu o\upsilon}-V_{\widehat{AB}}-V_{\widehat{A\Gamma}}$$

$$V_{\kappa o\lambda.\kappa\omega\nu o\upsilon}=\frac{\pi}{3}\left[13-(-3)\right]\left(4^2+4\cdot 16+16^2\right)=\pi\cdot\frac{16}{3}(16+64+256)=1792\pi\ \kappa.\mu.$$

$$V_{\widehat{AB}}=\pi\int_{-3}^{5}y^2dx=\pi\int_{-3}^{5}\left(25-x^2\right)dx=\pi\left[25x-\frac{x^3}{3}\right]_{-3}^{5}=\pi\left[125-\frac{125}{3}+75-\frac{27}{3}\right]=\frac{448\pi}{3}\ \kappa.\mu.$$

$$V_{\widehat{A\Gamma}}=\pi\int_{5}^{13}y^2dx=\pi\int_{5}^{13}\left(50x-225-x^2\right)dx=\pi\left[25x^2-225x-\frac{x^3}{3}\right]_{5}^{13}=$$

$$=\pi\left[4225-2925-\frac{2197}{3}-625+1125+\frac{125}{3}\right]=\pi\left[1800-\frac{2072}{3}\right]=\frac{3328\pi}{3}\kappa.\mu.$$

$$V_{\text{ZHT}}=1792\pi-\frac{448\pi}{3}-\frac{3328\pi}{3}=\frac{1600\pi}{3}\ \kappa.\mu.$$

4. $\dfrac{dy}{dx}-y\varepsilon\varphi x=-y^2\sigma\upsilon\nu x,\quad u=\dfrac{1}{y}\Rightarrow\dfrac{du}{dx}=\dfrac{du}{dy}\cdot\dfrac{dy}{dx}\Rightarrow\dfrac{du}{dx}=-\dfrac{1}{y^2}\cdot\dfrac{dy}{dx}$

$$\frac{dy}{dx}-y\varepsilon\varphi x=-y^2\sigma\upsilon\nu x\Rightarrow-\frac{1}{y^2}\frac{dy}{dx}+\frac{1}{y}\varepsilon\varphi x=\sigma\upsilon\nu x\Rightarrow\frac{du}{dx}+u\,\varepsilon\varphi x=\sigma\upsilon\nu x$$

$$I(x)=e^{\int P(x)dx}=e^{\int\varepsilon\varphi x dx}=e^{-\ln\sigma\upsilon\nu x}=\frac{1}{\sigma\upsilon\nu x}=\tau\varepsilon\mu x$$

$$\frac{du}{dx}\cdot\tau\varepsilon\mu x+u\cdot\tau\varepsilon\mu x\cdot\varepsilon\varphi x=\tau\varepsilon\mu x\cdot\sigma\upsilon\nu x\Rightarrow\frac{d}{dx}(u\cdot\tau\varepsilon\mu x)=1\Rightarrow\int d\left(u\cdot\tau\varepsilon\mu x\right)=\int 1dx\Rightarrow u\cdot\tau\varepsilon\mu x=x+c$$

$$\Rightarrow\frac{u}{\sigma\upsilon\nu x}=x+c\Rightarrow u=(x+c)\sigma\upsilon\nu x\Rightarrow\frac{1}{y}=(x+c)\sigma\upsilon\nu x\Rightarrow y=\frac{1}{(x+c)\sigma\upsilon\nu x}$$

y=1 όταν $\chi = 0 \Rightarrow 1 = \frac{1}{c} \Rightarrow c = 1$. Ειδική λύση: $y = \frac{1}{(x+1)\sigma\upsilon\nu x} \Rightarrow \boxed{y = \frac{\tau\varepsilon\mu x}{x+1}}$

$f(x) = \tau\varepsilon\mu x \Rightarrow f(0) = 1,\qquad f'(x) = \tau\varepsilon\mu x \cdot \varepsilon\phi x \Rightarrow f'(0) = 0$

$f''(x) = \tau\varepsilon\mu^3 x + \tau\varepsilon\mu x \cdot \varepsilon\phi^2 x \Rightarrow f''(0) = 1$

Άρα $\tau\varepsilon\mu x = 1 + \frac{x^2}{2} + \ldots$, $\quad (x+1)^{-1} = 1 - x + \frac{(-1)(-2)}{1\cdot 2}x^2 + \ldots = 1 - x + x^2 + \ldots$

$y = \tau\varepsilon\mu x (x+1)^{-1} = \left(1 + \frac{x^2}{2} + \ldots\right)\left(1 - x + x^2 + \ldots\right) = 1 - x + x^2 + \frac{x^2}{2} + \ldots = 1 - x + \frac{3}{2}x^2 + \ldots$

5. $\quad y = e^{2x}\eta\mu(x+a)$. Θα δείξω ότι ο τύπος ισχύει για ν=1

$y = e^{2x}\eta\mu(x+a) \Rightarrow \frac{dy}{dx} = 2e^{2x}\eta\mu(x+a) + e^{2x}\sigma\upsilon\nu(x+a) \Rightarrow$

$\frac{dy}{dx} = \sigma\phi\beta \cdot e^{2x}\eta\mu(x+a) + e^{2x}\sigma\upsilon\nu(x+a) \Rightarrow \frac{dy}{dx} = \frac{\sigma\upsilon\nu\beta}{\eta\mu\beta} \cdot e^{2x}\eta\mu(x+a) + e^{2x}\sigma\upsilon\nu(x+a) \Rightarrow$

$\frac{dy}{dx} = \frac{e^{2x}}{\eta\mu\beta} \cdot \left[\sigma\upsilon\nu\beta\,\eta\mu(x+a) + \eta\mu\beta\,\sigma\upsilon\nu(x+a)\right] \Rightarrow \frac{dy}{dx} = \frac{e^{2x}}{\eta\mu\beta} \cdot \eta\mu(\chi + \alpha + \beta)$.

Άρα ισχύει για ν=1

Υποθέτω ότι ισχύει για ν = κ δηλ. $\frac{d^\kappa y}{dx^\kappa} = \frac{e^{2x}}{\eta\mu^\kappa\beta}\eta\mu(x + a + \kappa\beta)$

Θα δείξω ότι ισχύει για ν = κ + 1

$$\frac{d^{\kappa+1} y}{dx^{\kappa+1}} = \frac{d}{dx}\left(\frac{d^\kappa y}{dx^\kappa}\right) = \frac{d}{dx}\left(\frac{e^{2x}}{\eta\mu^\kappa\beta}\eta\mu(x+a+\kappa\beta)\right) = \frac{2e^{2x}}{\eta\mu^\kappa\beta}\,\eta\mu(x+a+\kappa\beta) + \frac{e^{2x}}{\eta\mu^\kappa\beta}\,\sigma\upsilon\nu(x+a+\kappa\beta)$$

$$= \frac{e^{2x}}{\eta\mu^\kappa\beta}\frac{\eta\mu(x+a+\kappa\beta)\sigma\upsilon\nu\beta + \sigma\upsilon\nu(\chi+\alpha+\kappa\beta)\eta\mu\beta}{\eta\mu\beta} = \frac{e^{2x}}{\eta\mu^{\kappa+1}\beta}\eta\mu(x+a+\kappa\beta+\beta) =$$

$$= \frac{e^{2x}}{\eta\mu^{\kappa+1}\beta}\eta\mu\left(x+a+(\kappa+1)\beta\right) \Rightarrow \text{Ισχύει } \forall\, \nu \in \mathbb{N}^* \text{ δηλ.}$$

$$\frac{d^\nu y}{dx^\nu} = \frac{e^{2x}}{\eta\mu^\nu\beta}\eta\mu(x+a+\nu\beta), \quad \nu = 1,2,3,\ldots$$

β) $\eta\mu\alpha + \frac{\eta\mu(a+\beta)}{1!\,\eta\mu\beta}\left(\frac{\pi}{2}\right) + \frac{\eta\mu(a+2\beta)}{2!\,\eta\mu\beta}\left(\frac{\pi}{2}\right)^2 + \frac{\eta\mu(a+3\beta)}{3!\,\eta\mu\beta}\left(\frac{\pi}{2}\right)^3 + \ldots =$

$= f(0) + \frac{f'(0)}{1!}\cdot\frac{\pi}{2} + \frac{f''(0)}{2!}\cdot\left(\frac{\pi}{2}\right)^2 + \frac{f'''(0)}{3!}\left(\frac{\pi}{2}\right)^3 + \ldots = f\left(\frac{\pi}{2}\right) = e^\pi\eta\mu\left(\frac{\pi}{2}+\alpha\right) = e^\pi\sigma\upsilon\nu\alpha$