ΠΡΟΤΕΙΝΟΜΕΝΕΣ ΛΥΣΕΙΣ

Ζήτημα 1°

α)

$$\frac{2\eta\mu5\alpha\sigma\upsilon\nu3\alpha-\eta\mu8\alpha}{2\sigma\upsilon\nu8\alpha\sigma\upsilon\nu6\alpha-\sigma\upsilon\nu14\alpha}=\frac{\eta\mu(5\alpha+3\alpha)+\eta\mu(5\alpha-3\alpha)-\eta\mu8\alpha}{\sigma\upsilon\nu(8\alpha+6\alpha)+\sigma\upsilon\nu(8\alpha-6\alpha)-\sigma\upsilon\nu14\alpha}$$

$$=\frac{\cancel{\eta\mu8\alpha}+\eta\mu2\alpha-\cancel{\eta\mu8\alpha}}{\cancel{\sigma\upsilon\nu14\alpha}+\sigma\upsilon\nu2\alpha-\cancel{\sigma\upsilon\nu14\alpha}}=\frac{\eta\mu2\alpha}{\sigma\upsilon\nu2\alpha}\varepsilon\phi2\alpha$$

β) (i) $\varepsilon\phi(45°-\alpha)=\frac{\varepsilon\phi45°-\varepsilon\phi\alpha}{1+\varepsilon\phi45°\varepsilon\phi\alpha}=\frac{1-\varepsilon\phi\alpha}{1+\varepsilon\phi\alpha}=\frac{1-\frac{\eta\mu\alpha}{\sigma\upsilon\nu\alpha}}{1+\frac{\eta\mu\alpha}{\sigma\upsilon\nu\alpha}}=\frac{\frac{\sigma\upsilon\nu\alpha-\eta\mu\alpha}{\cancel{\sigma\upsilon\nu\alpha}}}{\frac{\sigma\upsilon\nu\alpha+\eta\mu\alpha}{\cancel{\sigma\upsilon\nu\alpha}}}=\frac{\sigma\upsilon\nu\alpha-\eta\mu\alpha}{\sigma\upsilon\nu\alpha+\eta\mu\alpha}$

(ii) $\varepsilon\phi(2\chi+15°)=\frac{\sigma\upsilon\nu\chi-\eta\mu\chi}{\sigma\upsilon\nu\chi+\eta\mu\chi}\Rightarrow\varepsilon\phi(2\chi+15°)=\varepsilon\phi(45°-\chi)\Rightarrow 2\chi+15°=180°\kappa+45°-\chi$

$\Rightarrow 3\chi=180°\kappa+30°\Rightarrow\chi=60°\kappa+10°.\quad\chi\in[0°,180°]$

$\kappa=0\Rightarrow\boxed{\chi=10°},\quad\kappa=1\Rightarrow\boxed{\chi=70°},\quad\kappa=2\Rightarrow\boxed{\chi=130°}$

γ) (i) $\frac{\eta\mu2\theta-\eta\mu\theta}{\sigma\upsilon\nu2\theta-\sigma\upsilon\nu\theta+1}=\frac{2\eta\mu\theta\sigma\upsilon\nu\theta-\eta\mu\theta}{2\sigma\upsilon\nu^2\theta\cancel{-1}-\sigma\upsilon\nu\theta\cancel{+1}}=\frac{\eta\mu\theta\cancel{(2\sigma\upsilon\nu\theta-1)}}{\sigma\upsilon\nu\theta\cancel{(2\sigma\upsilon\nu\theta-1)}}=\varepsilon\phi\theta$

(ii) $\frac{\eta\mu2\theta-\eta\mu\theta}{\sigma\upsilon\nu2\theta-\sigma\upsilon\nu\theta+1}=1\Rightarrow\varepsilon\phi\theta=1\Rightarrow\frac{2\cdot\varepsilon\phi\frac{\theta}{2}}{1-\varepsilon\phi^2\frac{\theta}{2}}=1\Rightarrow\varepsilon\phi^2\frac{\theta}{2}+2\varepsilon\phi\frac{\theta}{2}-1=0\Rightarrow$

$\varepsilon\phi\frac{\theta}{2}=\frac{-2\pm\sqrt{4+4}}{2}=\frac{-2\pm\sqrt{8}}{2}=\frac{-2\pm2\sqrt{2}}{2}=-1\pm\sqrt{2}$. Επειδή $0°<\theta<90°\Rightarrow 0°<\frac{\theta}{2}<45°$

$\Rightarrow\varepsilon\phi\frac{\theta}{2}=-1+\sqrt{2}$

Ζήτημα 2°

α) $\sigma\upsilon\nu2\chi+3\sigma\upsilon\nu\chi-1=0\Rightarrow 2\sigma\upsilon\nu^2\chi-1+3\sigma\upsilon\nu\chi-1=0\Rightarrow 2\sigma\upsilon\nu^2\chi+3\sigma\upsilon\nu\chi-2=0$,

$\sigma\upsilon\nu\chi=\omega,\quad 2\omega^2+3\omega-2=0\Rightarrow\omega_{1,2}=\frac{-3\pm5}{4}=\begin{cases}\omega_1=\frac{1}{2}\\ \omega_2=-2\end{cases}$

(I) $\sigma\upsilon\nu\chi=\frac{1}{2}=\sigma\upsilon\nu60°\Rightarrow\chi=360°\kappa\pm60°,\quad\kappa\in\mathbb{Z}$

(II) $\sigma\upsilon\nu\chi=-2$ αδύνατη εξίσωση.

β) (i) $\varepsilon\phi\theta+\sigma\phi\theta=\frac{\eta\mu\theta}{\sigma\upsilon\nu\theta}+\frac{\sigma\upsilon\nu\theta}{\eta\mu\theta}=\frac{\eta\mu^2\theta+\sigma\upsilon\nu^2\theta}{\eta\mu\theta\,\sigma\upsilon\nu\theta}=\frac{1}{\eta\mu\theta\,\sigma\upsilon\nu\theta}=\frac{2}{2\eta\mu\theta\,\sigma\upsilon\nu\theta}=\frac{2}{\eta\mu2\theta}$

(ii) $\varepsilon\phi\chi+\sigma\phi\chi=8\sigma\upsilon\nu2\chi\Rightarrow\frac{2}{\eta\mu2\chi}=8\sigma\upsilon\nu2\chi\Rightarrow 4\eta\mu2\chi\,\sigma\upsilon\nu2\chi=1\Rightarrow\eta\mu4\chi=\frac{1}{2}=\eta\mu30°$

$$\Rightarrow \begin{matrix} 4\chi = 360°\kappa + 30° \\ \underline{\chi = 90°\kappa + 7{,}5°} \quad \kappa \in \mathbb{Z} \end{matrix} \text{ ή } \begin{matrix} 4\chi = 360°\kappa + 180° - 30° \\ \underline{\chi = 90°\kappa + 37{,}5°} \quad \kappa \in \mathbb{Z} \end{matrix}$$

γ) $συν5\chi + συν3\chi = ημ3\chi + ημ\chi \Rightarrow$

$2συν4\chi\, συν\chi = 2ημ2\chi\, συν\chi \Rightarrow 2συνχ\left(συν4\chi - ημ2\chi\right) = 0 \Rightarrow$

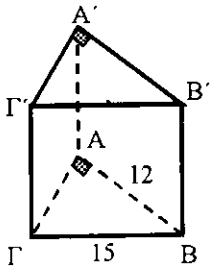
◊ $συνχ = 0 \Rightarrow \chi = 180°\kappa + 90,\ \kappa \in \mathbb{Z}$ για κ=0 $\Rightarrow \underline{\chi = 90°}$

◊ $συν4\chi - ημ2\chi = 0 \Rightarrow συν4\chi = ημ2\chi \Rightarrow συν4\chi = συν(90° - 2\chi) \Rightarrow 4\chi = 360°\kappa \pm (90° - 2\chi)$

✔ $4\chi = 360°\kappa - (90° - 2\chi) \Rightarrow$
$2\chi = 360°\kappa - 90° \Rightarrow$
$\chi = 180°\kappa - 45° \quad \kappa \in \mathbb{Z}$
$\kappa=1 \Rightarrow \underline{\chi = 135°}$

✔ $4\chi = 360°\kappa + (90° - 2\chi) \Rightarrow$
$6\chi = 360°\kappa + 90° \Rightarrow$
$\chi = 60°\kappa + 15° \quad \kappa \in \mathbb{Z}$
$\kappa=0 \Rightarrow \underline{\chi = 15°}$
$\kappa=1 \Rightarrow \underline{\chi = 75°}$
$\kappa=2 \Rightarrow \underline{\chi = 135°}$

Ζήτημα 3ο



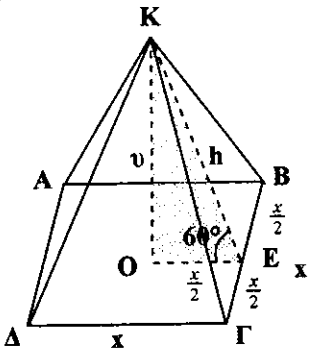
α) Π.Θ. $(AΓ)^2 = (BΓ)^2 - (AB)^2 \Rightarrow (AΓ)^2 = 15^2 - 12^2 \Rightarrow \underline{(AΓ) = 9\ cm}$

$E_{\pi} = \Pi_B \cdot \upsilon \Rightarrow E_{\pi} = (9+12+15)\cdot\upsilon \Rightarrow E_{\pi} = 36\upsilon,\quad E_{\Pi} = 360\,cm^2$

$\Rightarrow 36\upsilon = 360 \Rightarrow \boxed{\upsilon = 10\,cm}$. $V = E_B \cdot \upsilon \Rightarrow V = \frac{1}{2}\cdot 9\cdot 12\cdot 10 \Rightarrow \boxed{V = 540\,cm^3}$

β) $E_{\pi} = 8\alpha^2\ cm^2$.

$\widehat{KEO} = 60° \Rightarrow \widehat{OKE} = 30°,\ \widehat{KOE} = 90° \Rightarrow OE = \frac{KE}{2} \Rightarrow KE = h = \chi$



$E_{\pi} = \frac{1}{2}\Pi_B \cdot h \Rightarrow \frac{1}{2}\cdot 4\chi\cdot\chi = 8\alpha^2 \Rightarrow 4\chi^2 = 16\alpha^2 \Rightarrow \boxed{\chi = 2\alpha}$

$(KO)^2 = (KE)^2 - (OE)^2 \Rightarrow (KO)^2 = (2\alpha)^2 - \alpha^2 \Rightarrow (KO)^2 = 3\alpha^2 \Rightarrow$

$(KO) = \upsilon = \alpha\sqrt{3}\,cm$

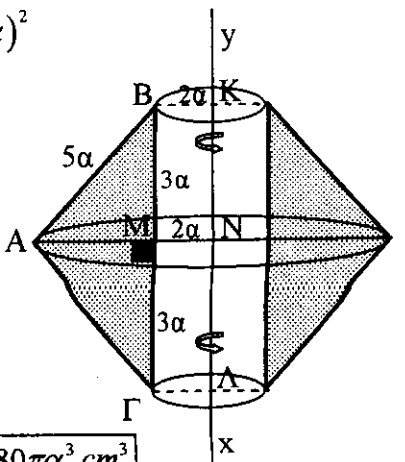
$V = \frac{1}{3}\cdot E_B \cdot \upsilon \Rightarrow V = \frac{1}{3}\cdot(2\alpha)^2\cdot\alpha\sqrt{3} \Rightarrow V = \frac{1}{3}\cdot 4\alpha^2\cdot\alpha\sqrt{3} \Rightarrow$

$$\boxed{V = \frac{4\alpha^3\sqrt{3}}{3}\,cm^3}$$

γ) π.θ. $A\hat{M}Γ: (AM)^2 = (AΓ)^2 - (ΓM)^2 \Rightarrow (AM)^2 = (5\alpha)^2 - (3\alpha)^2$

$\Rightarrow (AM)^2 = 25\alpha^2 - 9\alpha^2 \Rightarrow (AM)^2 = 16\alpha^2 \Rightarrow (AM) = 4\alpha$

(ΓΚ) =2α, (AN) = 6α, (AΓ) = 5α, (BΓ) = 6α, (ΓM) = 3α



$E_{Z\eta\tau.} = 2E_{E\Gamma} + E_{B\Gamma} \Rightarrow E_{Z\eta\tau.} = 2\pi(6\alpha + 2\alpha)\cdot 5\alpha + 2\pi\cdot 2\alpha\cdot 6\alpha \Rightarrow$

$E_{Z\eta\tau.} = 80\pi\alpha^2 + 24\pi\alpha^2 \Rightarrow \boxed{E_{Z\eta\tau.} = 104\pi\alpha^2\ cm^2}$

$V_{\zeta\eta\tau.} = 2V_{ANK\Gamma} - V_{BAK\Gamma} \Rightarrow$

$V_{\zeta\eta\tau.} = \frac{2\pi\cdot 3\alpha}{3}\left(4\alpha^2 + 12\alpha^2 + 36\alpha^2\right) - \pi\cdot 4\alpha^2\cdot 6\alpha \Rightarrow$

$V_{\zeta\eta\tau.} = 2\pi\alpha\cdot 52\alpha^2 - 24\pi\alpha^3 \Rightarrow V_{\zeta\eta\tau.} = 104\pi\alpha^3 - 24\pi\alpha^3 \Rightarrow \boxed{V_{\zeta\eta\tau.} = 80\pi\alpha^3\ cm^3}$

Ζήτημα 4ο

α) $E_{ολ.} = 2\alpha\beta + 2\beta\gamma + 2\gamma\alpha \Rightarrow$

$2200 = 60\cdot 20 + 60\cdot\gamma + 40\gamma \Rightarrow 2200 - 1200 = 100\gamma \Rightarrow 1000 = 100\gamma \Rightarrow \boxed{\gamma = 10\,cm}$

$V_{ορ.παραλ.} = 30\cdot 20\cdot 10 \Rightarrow \underline{V_{ορ.παραλ.} = 6000\,cm^3}$. $V_{κυβου} = \dfrac{6000}{750} \Rightarrow V_{κυβου} = 8\,cm^3$

$V_{κυβου} = \chi^3 \Rightarrow \chi^3 = 8 \Rightarrow \boxed{\chi = 2\,cm}$ το μήκος της ακμής του κύβου είναι 2 cm.

β) $\left.\begin{matrix} V_{σφ.} = \dfrac{4}{3}\pi R^3 \\ R = 1cm \end{matrix}\right\} \Rightarrow V_{σφ.} = \dfrac{4\pi}{3}\,cm^3,$

$\left.\begin{matrix} V_{χσφ} = \dfrac{4}{3}\pi\chi\,cm^3 \\ V_{νερου} = \pi\cdot 4^2\cdot 0,5 = 8\pi\,cm^3 \end{matrix}\right\} \Rightarrow \dfrac{4}{3}\pi\chi = 8\pi \Rightarrow 4\chi = 24 \Rightarrow \chi = 6$ σφαίρες.

γ) $(ΒΓ)^2 = 4^2 + 3^2 \Rightarrow (ΒΓ)^2 = 25 \Rightarrow (ΒΓ) = 5\,cm$

$E_{στερ.} = E_{\widehat{ΒΔ}} + E_{ΓΔ} + E_{ΒΓ} \Rightarrow$

$E_{στερ.} = 2\pi(ΑΔ)^2 + 2\pi(ΕΓ)(ΔΓ) + \pi(ΕΓ)(ΒΓ) \Rightarrow$

$E_{στερ.} = 2\pi\cdot 3^2 + 2\pi\cdot 3\cdot 7 + \pi\cdot 3\cdot 5 \Rightarrow$

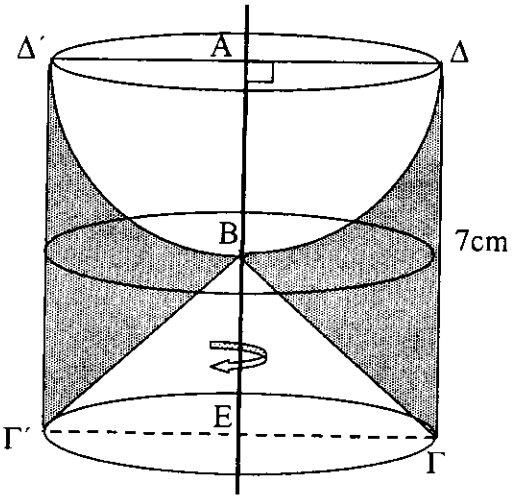
$E_{στερ.} = 18\pi + 42\pi + 15\pi \Rightarrow \boxed{E_{στερ.} = 75\pi\,cm^3}$

$V_{στερ.} = V_{κυλ.} - V_{ημισφ.} - V_{κων.} \Rightarrow$

$V_{στερ.} = \pi(ΕΓ)^2\cdot(ΔΓ) - \dfrac{2}{3}\pi(ΑΔ)^3 - \dfrac{1}{3}\pi(ΕΓ)^2\cdot(ΒΕ)$

$V_{στερ.} = \pi\cdot 9\cdot 7 - \dfrac{2}{3}\pi\cdot 27 - \dfrac{1}{3}\pi\cdot 9\cdot 4 \Rightarrow$

$V_{στερ.} = 63\pi + 8\pi - 12\pi \Rightarrow \boxed{V_{στερ.} = 33\pi\,cm^3}$



Ζήτημα 5ο

α) $y = 3συν2\chi + 5ημ2\chi \Rightarrow \dfrac{dy}{dx} = -6ημ2\chi + 10συν2\chi \Rightarrow \dfrac{d^2y}{dx^2} = -12συν2\chi - 20ημ2\chi$

Άρα $\dfrac{d^2y}{dx^2} + 4y = -12συν2\chi - 20ημ2\chi + 4(3συν2\chi + 5ημ2\chi) =$

$= \cancel{-12συν2\chi} \cancel{-20ημ2\chi} \cancel{+12συν2\chi} + \cancel{20ημ2\chi} = 0$

β) $\int 14συν8\chi συν4\chi\,dx = 7\int[συν(8\chi - 4\chi) + συν(8\chi + 4\chi)]\,dx =$

$= 7\int[συν4\chi + συν12\chi]dx = \dfrac{7}{4}ημ4\chi + \dfrac{7}{12}ημ12\chi + c$

γ) (i) $y=\dfrac{\alpha x}{x^2+1}$, $P\left(\dfrac{1}{2},\dfrac{8}{5}\right)\Rightarrow \dfrac{8}{5}=\dfrac{\alpha\cdot\frac{1}{2}}{\left(\frac{1}{2}\right)^2+1}\Rightarrow \dfrac{\alpha}{2}=\dfrac{8}{5}\cdot\dfrac{5}{4}\Rightarrow \dfrac{\alpha}{2}=2\Rightarrow \boxed{\alpha=4}$

(ii) $y=\dfrac{4x}{x^2+1}\Rightarrow \dfrac{dy}{dx}=\dfrac{4\left(\chi^2+1\right)-4\chi\cdot 2\chi}{\left(\chi^2+1\right)^2}\Rightarrow \dfrac{dy}{dx}=\dfrac{4\chi^2+4-8\chi^2}{\left(\chi^2+1\right)^2}\Rightarrow \dfrac{dy}{dx}=\dfrac{-4\chi^2+4}{\left(\chi^2+1\right)^2}\Rightarrow$

$\dfrac{dy}{dx}=\dfrac{-4(\chi-1)(\chi+1)}{\left(\chi^2+1\right)^2}$, $\dfrac{dy}{dx}=0\Rightarrow \chi=1$ ή $x=-1$

χ	$-\infty$		-1		1		$+\infty$
$\frac{dy}{dx}$		$-$	0	$+$	0	$-$	
ψ		$\searrow$	min	$\nearrow$	max	$\searrow$	

$\chi=-1\Rightarrow \psi_{\min}=-2\Rightarrow \min(-1,-2)$

$\chi=1\Rightarrow \psi_{\min}=2\Rightarrow \max(1,2)$

Ζήτημα 6ο

$I_1=\int_1^4\left(6x^2-\dfrac{2}{\sqrt{x}}+2\right)dx=I_1=\int_1^4\left(6x^2-2x^{-\frac{1}{2}}+2\right)dx=\left[2x^3-\dfrac{2x^{\frac{1}{2}}}{\frac{1}{2}}+2x\right]_1^4=$

$=\left[2x^3-4x^{\frac{1}{2}}+2x\right]_1^4=\left(2\cdot 4^3-4\sqrt{4}+2\cdot 4\right)-\left(2\cdot 1^3-4\sqrt{1}+2\cdot 1\right)=(128-8+8)-(2-4+2)=$

$=128$

$I_2=\int_0^{\frac{\pi}{2}}\sigma\upsilon\nu^2 x\,dx=\dfrac{1}{2}\int_0^{\frac{\pi}{2}}(1+\sigma\upsilon\nu 2\chi)dx=\dfrac{1}{2}\left[\chi+\dfrac{1}{2}\eta\mu 2\chi\right]_0^{\frac{\pi}{2}}=\dfrac{1}{2}\left[\left(\dfrac{\pi}{2}+\dfrac{1}{2}\eta\mu 2\cdot\dfrac{\pi}{2}\right)-\left(0+\dfrac{1}{2}\eta\mu 2\cdot 0\right)\right]$

$=\dfrac{\pi}{4}$

$\lambda_{\varepsilon\alpha\upsilon}=-\dfrac{1}{9}\Rightarrow \lambda_{\varepsilon\phi}=9$

$y=2ax^2-3x+2\Rightarrow \dfrac{dy}{dx}=4ax-3$, $P(1,\kappa)\Rightarrow \lambda_{\varepsilon\phi}=\left.\dfrac{dy}{dx}\right|_{\chi=1}=4\alpha\cdot 1-3\Rightarrow 9=4a-3\Rightarrow$

$4a=12\Rightarrow \boxed{a=3}$

$P(1,\kappa)$ ανήκει στην καμπύλη $y=2ax^2-3x+2\Leftrightarrow \kappa=2\cdot 3\cdot 1^2-3\cdot 1+2\Rightarrow \boxed{\kappa=5}\Rightarrow P(1,5)$

Εξίσωση εφαπτομένης στο $P(1,5)$, $\lambda_{\varepsilon\phi}=9\Rightarrow \psi-5=9(\chi-1)\Rightarrow \boxed{\psi=9\chi-4}$

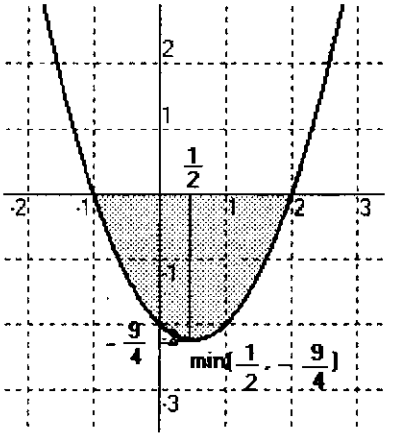
$y=x^2-x-2$, $\chi\in\mathbb{R}$. Για $\chi=0\Rightarrow \psi=-2\Rightarrow(0,2)$.

Για $\psi=0\Rightarrow(\chi-2)(\chi+1)=0\Rightarrow \chi=2$, $\chi=-1\Rightarrow(-1,0)$, $(2,0)$

$\dfrac{dy}{dx}=2\chi-1$, $\dfrac{dy}{dx}=0\Rightarrow 2\chi-1=0\Rightarrow \chi=\dfrac{1}{2}$

Για $\chi=\dfrac{1}{2}\Rightarrow \psi_{\min}=-\dfrac{9}{4}\Rightarrow \min\left(\dfrac{1}{2},-\dfrac{9}{4}\right)$

χ	$-\infty$		$\frac{1}{2}$		$+\infty$
$\frac{dy}{dx}$		$-$	0	$+$	
ψ		$\searrow$	min	$\nearrow$	



(ii) $E=-\int_{-1}^{2}\left(x^2-x-2\right)dx \Rightarrow E=-\left[\frac{x^3}{3}-\frac{x^2}{2}-2x\right]_{-1}^{2}$

$E=-\left[\left(\frac{8}{3}-2-4\right)-\left(-\frac{1}{3}-\frac{1}{2}+2\right)\right] \Rightarrow E=-\left(3-8+\frac{1}{2}\right)$

$E=5-\frac{1}{2} \Rightarrow E=\frac{9}{2}\ \tau.\mu ov.$