ΠΡΟΤΕΙΝΟΜΕΝΕΣ ΛΥΣΕΙΣ

1. $y = 3e^{2x} - 5e^{-2x} \Rightarrow \dfrac{dy}{dx} = 6e^{2x} + 10e^{-2x} \Rightarrow \dfrac{d^2y}{dx^2} = 12e^{2x} - 20e^{-2x} \Rightarrow \dfrac{dy^2}{dx^2} = 4\underbrace{\left(3e^{2x} - 5e^{-2x}\right)}_{\psi}$

$$\Rightarrow \frac{dy^2}{dx^2} = 4y \Rightarrow \frac{dy^2}{dx^2} - 4y = 0$$

2.

Αξία	*Κέρδος*	*Πώληση*
£100	*£15*	*£115*
χ;		*£2990*

$$\frac{100}{\chi} = \frac{115}{2990} \Rightarrow \chi = \frac{100 \cdot 2990}{115} \Rightarrow \chi = 2600$$

Το αυτοκίνητο το αγόρασε ***£2600***

3.

Μισθός x_i	Αριθμός υπαλλήλων f_i	$f_i \cdot \chi_i$	$(\chi_i - \bar{\chi})^2$	$f_i \cdot (\chi_i - \bar{\chi})^2$
60	3	180	40^2=1600	$3 \cdot 1600 = 4800$
80	5	400	400	2000
110	6	660	100	600
120	4	480	400	1600
140	2	280	1600	3200
	$\sum fi = 20$	$\sum f_i \chi_i = 2000$		$\sum fi(\chi_i - \bar{\chi})^2 = 12200$

$$\bar{\chi} = \frac{\sum f_i \chi_i}{\sum f_i} = \frac{2000}{20} = 100, \qquad \sigma = \sqrt{\frac{\sum fi(\chi_i - \bar{\chi})^2}{\sum f_i}} = \sqrt{\frac{12\,200}{20}} = \sqrt{610} = 24{,}69$$

4. *Έστω ότι η εξίσωση του ζητούμενου κύκλου είναι* $\chi^2 + \psi^2 + 2g\chi + 2f\psi + c = 0$

$K(-g,-f)$ *και* $R = \sqrt{g^2 + f^2 - c}$,

(ε): $\psi = -\chi$ $\quad K \in (\varepsilon)$, $\chi = -g$, $\psi = -f \Rightarrow -f = g$ *(1)*

$\chi^2 + \psi^2 - 2\chi + 10\psi - 24 = 0$

$\underline{\chi^2 + \psi^2 + 2\chi + 2\psi - 8 = 0}$ $\quad (-)$

$-4\chi + 8\psi - 16 = 0 \Rightarrow \chi - 2\psi + 4 = 0 \Rightarrow \chi = 2\psi - 4 \quad (1)$

$(2\psi - 4)^2 + \psi^2 - 2(2\psi - 4) + 10\psi - 24 = 0 \Rightarrow$

$4\psi^2 - 16\psi + 16 + \psi^2 - 4\psi + 8 + 10\psi - 24 = 0 \Rightarrow 5\psi^2 - 10\psi = 0 \Rightarrow \psi(\psi - 2) = 0 \Rightarrow$

ψ=0 $\overset{(1)}{\Rightarrow}$ *χ=–4 ή ψ =2* $\overset{(1)}{\Rightarrow}$ *χ=0* $\Rightarrow$ *Σημεία τομής των δύο κύκλων Α(-4,0) και Β(0,2)*
Τα σημεία Α και Β ανήκουν στον ζητούμενο κύκλο άρα οι συντεταγμένες των σημείων επαληθεύουν την εξίσωση του.

$$\left.\begin{matrix}(-4)^2+0^2-8g+0+c=0\\ 0^2+2^2+0+4f+c=0\end{matrix}\right\}\Rightarrow\left.\begin{matrix}16-8g+c=0\\ 4+4f+c=0\\ g=-f\end{matrix}\right\}\Rightarrow\left.\begin{matrix}16+8f+c=0\\ 4+4f+c=0\\ g=-f\end{matrix}\right\}\Rightarrow$$

$$12+4f=0\Rightarrow\boxed{f=-3}\Rightarrow\boxed{g=3}\Rightarrow\boxed{c=8}\Rightarrow\boxed{\chi^2+\psi^2+6\chi-6\psi+8=0}$$

2^{ος} τρόπος ("δέσμη κύκλων")

Οι κύκλοι που περνούν από τα σημεία τομής των δοθέντων κύκλων είναι :

$$\chi^2+\psi^2-2\chi+10\psi-24+\mu\cdot(\chi^2+\psi^2+2\chi+2\psi-8)=0\Rightarrow$$

$$(\mu+1)\chi^2+(\mu+1)\psi^2+2(-1+\mu)\chi+2(5+\mu)\psi+(-24-8\mu)=0\Rightarrow$$

$$\chi^2+\psi^2+2\frac{\mu-1}{\mu+1}\chi+2\frac{\mu+5}{\mu+1}\psi-\frac{8\mu+24}{\mu+1}=0\quad(1)\Rightarrow \mathrm{K}\left(-\frac{\mu-1}{\mu+1},-\frac{\mu+5}{\mu+1}\right),$$

(ε): $\psi=-\chi$ $\quad K\in(\varepsilon)\Rightarrow -\frac{\mu+5}{\mu+1}=\frac{\mu-1}{\mu+1},\quad \mu\neq-1\Rightarrow -(\mu+5)=\mu-1\Rightarrow\mu=-2$

$$\overset{(1)}{\Rightarrow}\chi^2+\psi^2+2\cdot\frac{-3}{-1}\chi+2\cdot\frac{3}{-1}\psi-\frac{-16+24}{-1}=0\Rightarrow\boxed{\chi^2+\psi^2+6\chi-6\psi+8=0}$$

5. (α) $\chi^2\psi\frac{d\psi}{d\chi}=\ln\chi\Rightarrow\psi\,d\psi=\chi^{-2}\ln\chi\,d\chi\Rightarrow\int\psi\,d\psi=\int\chi^{-2}\ln\chi\,d\chi\Rightarrow$

$$\frac{\psi^2}{2}=\int\ln\chi\,d(-\chi^{-1})\Rightarrow\frac{\psi^2}{2}=-\chi^{-1}\ln\chi-\int-\chi^{-1}d(\ln\chi)\Rightarrow\frac{\psi^2}{2}=-\frac{\ln\chi}{\chi}+\int\frac{1}{\chi}\cdot\frac{1}{\chi}d\chi$$

$$\Rightarrow\frac{\psi^2}{2}=-\frac{\ln\chi}{\chi}+\int\chi^{-2}d\chi\Rightarrow\frac{\psi^2}{2}=-\frac{\ln\chi}{\chi}+(-\chi^{-1})+c\Rightarrow\boxed{\frac{\psi^2}{2}=-\frac{\ln\chi}{\chi}-\frac{1}{\chi}+c}.$$

(β) ψ=2, χ=1 $\Rightarrow\frac{2^2}{2}=-\frac{\ln 1}{1}-\frac{1}{1}+c\Rightarrow 2=0-1+c\Rightarrow c=3$

$$\frac{\psi^2}{2}=-\frac{\ln\chi}{\chi}-\frac{1}{\chi}+3\Rightarrow\boxed{\psi^2=-\frac{2\ln\chi}{\chi}-\frac{2}{\chi}+6}$$ *Ειδική λύση.*

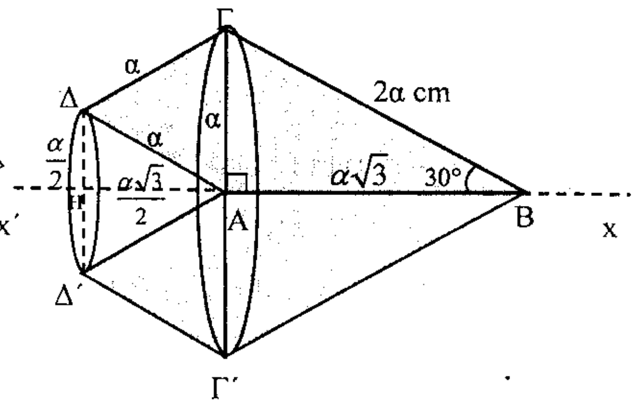
6. $V = V_{(\mathrm{AB\Gamma})} + V_{\mathrm{A\Gamma\Delta H}} - V_{(\mathrm{A\Delta H})}$

$(\overset{\Delta}{\mathrm{AB\Gamma}})$ *ορθογώνιο*, $\hat{\mathrm{B}} = 30° \Rightarrow$

$\mathrm{A\Gamma} = \dfrac{\mathrm{B\Gamma}}{2} \Rightarrow A\Gamma = \alpha$

$(AB)^2 = (B\Gamma)^2 - (A\Gamma)^2 \Rightarrow (AB)^2 = 4\alpha^2 - \alpha^2 \Rightarrow$

$(AB)^2 = 3\alpha^2 \Rightarrow AB = \alpha\sqrt{3}$



$\left(\overset{\Delta}{\mathrm{A\Delta H}}\right)$ *ορθογώνιο*, $\widehat{\mathrm{\Delta AH}} = 30°$, $A\Delta = \alpha$

$\Rightarrow \Delta H = \dfrac{\alpha}{2}, \quad AH = \dfrac{\alpha\sqrt{3}}{2}$

$$V_{o\lambda} = \frac{1}{3}\pi(\mathrm{A\Gamma})^2 \cdot (\mathrm{AB}) + \frac{\pi(\mathrm{HA})}{3}\left[(\mathrm{A\Gamma})^2 + (\mathrm{A\Gamma})(\mathrm{\Delta H}) + (\mathrm{\Delta H})^2\right] - \frac{1}{3}\pi(\mathrm{\Delta H})^2 \cdot (\mathrm{HA})$$

$$V_{o\lambda} = \frac{\pi\left(\dfrac{\alpha\sqrt{3}}{2}\right)}{3}\left[\alpha^2 + \alpha\cdot\frac{\alpha}{2} + \left(\frac{\alpha}{2}\right)^2\right] - \frac{1}{3}\pi\left(\frac{\alpha}{2}\right)^2\frac{\alpha\sqrt{3}}{2} + \frac{1}{3}\pi\alpha^2\left(\alpha\sqrt{3}\right)$$

$$V_{o\lambda} = \frac{\pi\alpha^3\sqrt{3}}{3} + \frac{7\pi\alpha^3\sqrt{3}}{24} - \frac{\pi\alpha^3\sqrt{3}}{24} \Rightarrow \boxed{V_{o\lambda} = \frac{7\pi\alpha^3\sqrt{3}}{12}\ cm^3}$$

7. $2\ \text{Τοξσυν}\,\chi = \text{Τοξημ}\,\chi \quad (1)$

Θέτουμε $\text{τοξσυν}\chi = \alpha \Rightarrow \text{συν}\alpha = \chi, \quad 0 \leq \alpha \leq \pi$

$\text{τοξημ}\chi = \beta \Rightarrow \text{ημ}\beta = \chi, \quad -\dfrac{\pi}{2} \leq \beta \leq \dfrac{\pi}{2}$

$(1) \Rightarrow 2\alpha = \beta \Rightarrow \text{συν}2\alpha = \text{συν}\beta \Rightarrow 2\text{συν}^2\alpha - 1 = +\sqrt{1-\text{ημ}^2\beta} \qquad (\text{συν}\beta \geq 0) \Rightarrow$

$2\chi^2 - 1 = \sqrt{1-\chi^2} \Rightarrow (2\chi^2-1)^2 = 1-\chi^2 \Rightarrow 4\chi^4 - 3\chi^2 = 0 \Rightarrow \chi^2(4\chi^2-3) = 0 \Rightarrow$

$\Rightarrow \chi^2(2\chi-\sqrt{3})(2\chi+\sqrt{3}) = 0 \Rightarrow \chi = 0 \quad \text{ή} \quad \Rightarrow \chi = \pm\dfrac{\sqrt{3}}{2}$

<u>Επαλήθευση</u>

(i) $\chi = 0 \Rightarrow 2\,\text{τοξσυν}0 \overset{;}{=} \text{τοξημ}0 \Rightarrow 2\cdot\dfrac{\pi}{2} \neq 0 \Rightarrow \chi = 0$ *Απορρίπτεται*

(ii) $\chi = -\dfrac{\sqrt{3}}{2} \Rightarrow 2\text{τοξσυν}\left(-\dfrac{\sqrt{3}}{2}\right) \overset{;}{=} \text{τοξημ}\left(-\dfrac{\sqrt{3}}{2}\right) \Rightarrow 2\left(-\dfrac{5\pi}{6}\right) \neq -\dfrac{\pi}{3}$

Άρα $\chi = -\dfrac{\sqrt{3}}{2}$ *απορρίπτεται.*

(iii) $\chi = \dfrac{\sqrt{3}}{2} \Rightarrow 2\,\text{τοξσυν}\dfrac{\sqrt{3}}{2} \overset{;}{=} \text{τοξημ}\dfrac{\sqrt{3}}{2} \Rightarrow 2\cdot\dfrac{\pi}{6} = \dfrac{\pi}{3}$ *Ισχύει. Άρα* $\boxed{\chi = \dfrac{\sqrt{3}}{2}}$ *Δεκτή*

8. $A=\begin{pmatrix}1 & 3\\ 2 & -1\end{pmatrix}\Rightarrow$

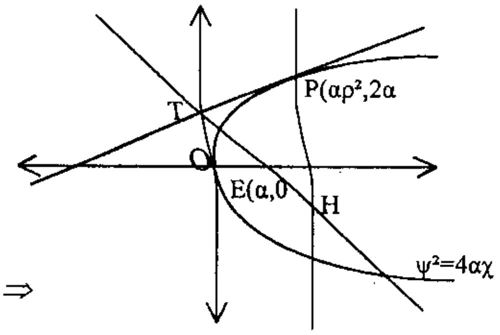
$$A^2=\begin{pmatrix}1 & 3\\ 2 & -1\end{pmatrix}\begin{pmatrix}1 & 3\\ 2 & -1\end{pmatrix}=\begin{pmatrix}1\cdot1+3\cdot2 & 1\cdot3+3\cdot(-1)\\ 2\cdot1+(-1)\cdot2 & 2\cdot3+(-1)\cdot(-1)\end{pmatrix}=\begin{pmatrix}7 & 0\\ 0 & 7\end{pmatrix}=7\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=7\cdot I$$

$A^4=A^2\cdot A^2=(7\cdot I)\cdot(7\cdot I)=7^2\cdot I^2=7^2\cdot I,\qquad A^{20}=\left(A^2\right)^{10}=(7\cdot I)^{10}=7^{10}\cdot I^{10}=7^{10}\cdot I$

$$A^{20}+\mu\cdot A^4+7\nu\cdot I=(O)\Rightarrow 7^{10}\cdot I+\mu\cdot 7^2\cdot I+7\nu\cdot I=(O)\Rightarrow\left(7^{10}+7^2\mu+7\nu\right)\cdot I=(O)\Rightarrow$$

$$7^{10}+7^2\mu+7\nu=0\Rightarrow 7^9+7\mu+\nu=0\Rightarrow\boxed{7\mu+\nu=-7^9}$$

9. *(α)* $\psi^2=4\alpha\chi\Rightarrow 2\psi\dfrac{d\psi}{d\chi}=4\alpha\Rightarrow\dfrac{d\psi}{d\chi}=\dfrac{2\alpha}{\psi}\Rightarrow\lambda_{\varepsilon\phi}=\dfrac{\cancel{2\alpha}}{\cancel{2\alpha}\rho}\Rightarrow\lambda_{\varepsilon\phi}=\dfrac{1}{\rho}$



εφ: $\psi-\psi_1=\lambda(\chi-\chi_1)\Rightarrow\psi-2\alpha\rho=\dfrac{1}{\rho}(\chi-\alpha\rho^2)$

$\psi\rho-2\alpha\rho^2=\chi-\alpha\rho^2\Rightarrow\boxed{\rho\psi=\chi+\alpha\rho^2}$ *(1)*

(β) $\boxed{T}$ $\chi=0\overset{(1)}{\Rightarrow}\rho\psi=0+\alpha\rho^2\Rightarrow\psi=\alpha\rho\Rightarrow T(0,\alpha\rho)$

Εξίσωση PH: $\chi=\alpha\rho^2$,

Εξίσωση TE: $T(0,\alpha\rho)$, $E(\alpha,0)$ $\quad\dfrac{\psi-\psi_1}{\chi-\chi_1}=\dfrac{\psi_2-\psi_1}{\chi_2-\chi_1}\Rightarrow$

$$\frac{\psi-\alpha\rho}{\chi-0}=\frac{0-\alpha\rho}{\alpha-0}\Rightarrow\frac{\psi-\alpha\rho}{\chi}=\frac{-\rho}{1}\Rightarrow\psi-\alpha\rho=-\rho\chi\Rightarrow\boxed{\rho\chi+\psi=\alpha\rho}\ (TE)$$

$$\boxed{H}\quad\left.\begin{aligned}\chi&=\alpha\rho^2\\ \rho\chi+\psi&=\alpha\rho\end{aligned}\right\}\Rightarrow\alpha\rho^3+\psi=\alpha\rho\Rightarrow\psi=\alpha\rho(1-\rho^2)\Rightarrow H\left(\alpha\rho^2,\alpha\rho(1-\rho^2)\right)$$

$$\left.\begin{aligned}\chi&=\alpha\rho^2\\ \psi&=\alpha\rho(1-\rho^2)\end{aligned}\right\}\Rightarrow\left.\begin{aligned}\rho^2&=\frac{\chi}{\alpha}\\ \psi^2&=\alpha^2\rho^2(1-\rho^2)^2\end{aligned}\right\}\Rightarrow\psi^2=\alpha\cancel{{}^2}\frac{\chi}{\cancel{\alpha}}\left(1-\frac{\chi}{\alpha}\right)^2$$

$$\Rightarrow\psi^2=\alpha\chi\frac{(\alpha-\chi)^2}{\alpha^2}\Rightarrow\boxed{\psi^2=\frac{\chi(\alpha-\chi)^2}{\alpha}}$$ *Γεωμετρικός τόπος του Η*

10. *0,3,4,5,6,7* *(α) Πενταψήφιοι*

i. Αρχίζει με 5,7 και τελειώνει με 0,4,6

2	4	3	2	3

= 144 αριθμοί

ii. Αρχίζει με 6 και τελειώνει σε 0,4

1	4	3	2	2

= 48 αριθμοί

(β) Εξαψήφιοι

i. Αρχίζει με 4,6,1 και τελειώνει με 4,6

2	4	3	2	1	2

= 96 αριθμοί

ii. Αρχίζει με 3,5,6 και τελειώνει σε 0,4,6

3	4	3	2	1	3

= 216 αριθμοί

Συνολικά 144+48+96+216 = 504 αριθμοί.

ΜΕΡΟΣ Β΄

1. $\psi = \dfrac{\chi^2 - 2\chi + 2}{\chi^2 - 2\chi}$

(α) *Πεδίο ορισμού:* $\chi^2\text{-}2\neq 0 \Rightarrow \chi(\chi-2)\neq 0 \Rightarrow \chi\neq 0$ *και* $\chi\neq 2 \Rightarrow \chi \in \mathbb{R} - \{0,2\}$

$$\frac{d\psi}{d\chi} = \frac{(2\chi-2)(\chi^2-2\chi)-(2\chi-2)(\chi^2-2\chi+2)}{(\chi^2-2\chi)^2} \Rightarrow$$

$$\frac{d\psi}{d\chi} = \frac{(2\chi-2)\left[\chi^2-2\chi-\chi^2+2\chi-2\right]}{\chi^2(\chi-2)^2} \Rightarrow \frac{d\psi}{d\chi} = \frac{-4(\chi-1)}{\chi^2(\chi-2)^2}$$

$$\frac{d\psi}{d\chi} = 0 \Rightarrow \frac{-4(\chi-1)}{\chi^2(\chi-2)^2} = 0 \Rightarrow \chi = 1 \;,\; \chi\neq 0 \text{ και } \chi\neq 2$$

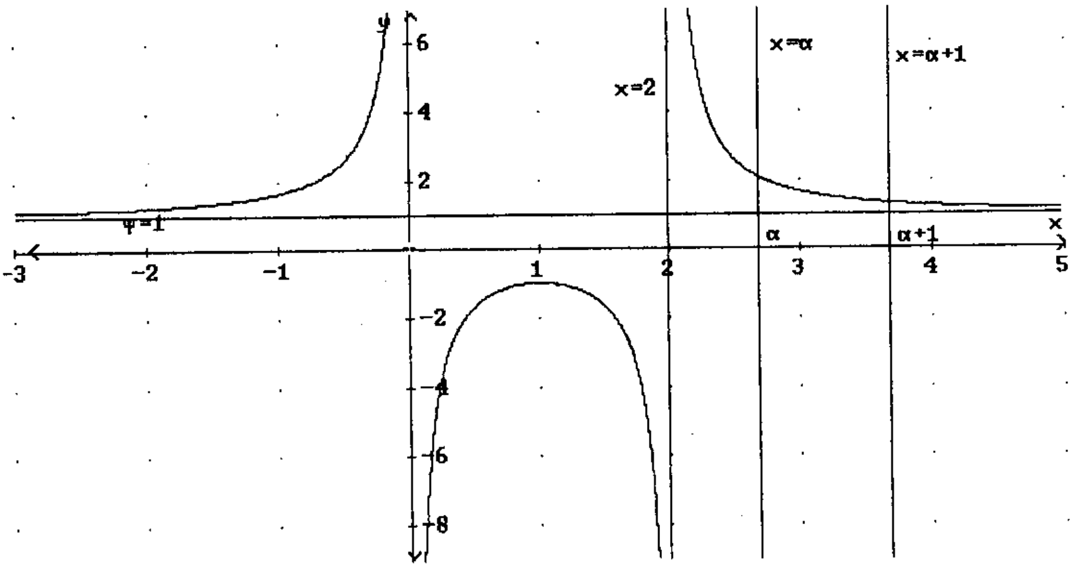
χ	$-\infty$	0		1		2	$+\infty$
$\frac{d\psi}{d\chi}$	+	‖	+	0	−	‖	−
ψ	↗	‖	↗		↘	‖	↘

Για $\chi=1 \Rightarrow \psi_{max} = \dfrac{1-2+2}{1-2} = -1$

$\Rightarrow$ ***max(1,–1)***

Κατακόρυφη ασύμπτωτη: $\chi = 0$ *και* $\chi = 2$

Οριζόντια ασύμπτωτη: $L = \lim\limits_{\chi\to\pm\infty} \dfrac{\chi^2-2\chi+2}{\chi^2-2\chi} = 0 \Rightarrow$ *Κατακόρυφη ασύμπτωτη:* $\psi = 0$



$$E = \int_{\alpha}^{\alpha+1} \frac{\chi^2-2\chi+2}{\chi^2-2\chi}\,d\chi = \int_{\alpha}^{\alpha+1}\left(1+\frac{2}{\chi^2-2\chi}\right)d\chi = \int_{\alpha}^{\alpha+1}\left(1+\frac{2}{\chi(\chi-2)}\right)d\chi$$

$$\frac{2}{\chi(\chi-2)} = \frac{A}{\chi} + \frac{B}{\chi-2} \Rightarrow 2 \equiv A(\chi-2)+B\chi \Rightarrow A = -1 \;,\; B = 1$$

$$E = \int_{\alpha}^{\alpha+1}\left(1-\frac{1}{\chi}+\frac{1}{\chi-2}\right)d\chi = \left[\chi - \ln|\chi| + \ln|x-2|\right]_{\alpha}^{\alpha+1}$$

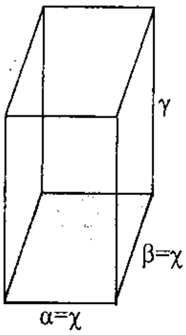
$$E = \left[\alpha+1-\ln(a+1)+\ln(a-1)\right]-\left[a-\ln a+\ln(a-2)\right]$$

$$E = \cancel{a}+1-\ln(a+1)+\ln(a-1)-\cancel{a}+\ln a-\ln(a-2) \Rightarrow E = 1+\ln\frac{a(a-1)}{(a+1)(a-2)}$$

$$\cancel{1}+\ln\frac{3}{2} = \cancel{1}+\ln\frac{a(a-1)}{(a+1)(a-2)} \Rightarrow \frac{3}{2} = \frac{a(a-1)}{(a+1)(a-2)} \Rightarrow 3(a+1)(a-2)=2a(a-1) \Rightarrow$$

$3\alpha^2-3\alpha-6=2\alpha^2-2\alpha \Rightarrow \alpha^2-\alpha-6=0 \Rightarrow (\alpha-3)(\alpha-2)=0 \Rightarrow \boxed{\alpha=3}$ ή $\alpha=-2$ απορρίπτεται $(\alpha>2)$

2.



$\alpha=\beta=\chi, \quad V = 10000\,m^3$

$V=\alpha\cdot\beta\cdot\gamma \Rightarrow 10000 = \chi\cdot\chi\cdot\gamma \Rightarrow \gamma = \dfrac{10000}{\chi^2}$

$E_B = \chi^2 \;,\; E_\Pi = 4\chi\cdot\gamma \Rightarrow E_\Pi = 4\chi\dfrac{10000}{\chi^2} \Rightarrow E_\Pi = \dfrac{40000}{\chi}$

Κόστος $\psi = 5\cdot E_B + 2\cdot E_B \Rightarrow \boxed{\psi = 5\chi^2+\dfrac{80000}{\chi}}$

$$\left.\begin{array}{l}\dfrac{d\psi}{d\chi} = 10\chi-\dfrac{80000}{\chi^2}\\ \dfrac{d\psi}{d\chi}=0\end{array}\right\} \Rightarrow 10\chi-\frac{80000}{\chi^2}=0 \Rightarrow 10\chi^3-80000=0 \Rightarrow$$

$10\chi^3=80000 \Rightarrow \chi^3=8000 \Rightarrow \chi=20\ m$

$\dfrac{d^2\psi}{d\chi^2} = 10+\dfrac{160000}{\chi^3} > 0 \Rightarrow$ για $\chi=20$ η συνάρτηση παρουσιάζει ελάχιστο.

$\Rightarrow \alpha = 20\ m\,,\ \beta = 20\ m\ ,\ \gamma = 25\ m\,.$

3. *(α)* $\displaystyle\int\frac{2\chi-1}{4\chi^2+9}d\chi = \int\frac{2\chi}{4\chi^2+9}d\chi - \int\frac{1}{4\chi^2+9}d\chi$

$$= \int\frac{(4\chi^2+9)'}{4\chi^2+9}d\chi - \frac{1}{9}\int\frac{1}{\left(\frac{2\chi}{3}\right)^2+1}d\chi = \frac{1}{4}\ln(4\chi^2+9)-\frac{1}{9}\frac{1}{\frac{2}{3}}\tau o\xi\varepsilon\varphi\left(\frac{2\chi}{3}\right)+c$$

$$= \boxed{\frac{1}{4}\ln(4\chi^2+9)-\frac{1}{6}\tau o\xi\varepsilon\varphi\left(\frac{2\chi}{3}\right)+c}$$

(β) $\displaystyle \chi\frac{d\psi}{d\chi}-\psi = \frac{\chi^2(2\chi-1)}{4\chi^2+9} \Rightarrow \frac{d\psi}{d\chi}-\frac{1}{\chi}\psi = \frac{\chi(2\chi-1)}{4\chi^2+9} \qquad (\chi>0)$

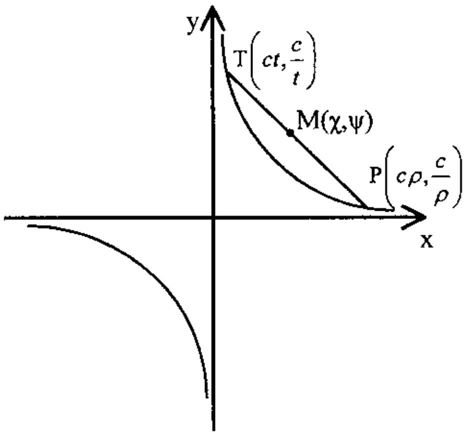
Παράγοντας Ολοκλήρωσης $\mathrm{I}(\chi)=e^{\int -\frac{1}{\chi}d\chi}=e^{-\ln\chi}=\dfrac{1}{\chi}$

$$\Rightarrow \frac{1}{\chi}\cdot\frac{d\psi}{d\chi}-\frac{1}{\chi^2}\cdot\psi=\frac{2\chi-1}{4\chi^2+9} \Rightarrow \frac{d\left(\frac{1}{\chi}\cdot\psi\right)}{d\chi}=\frac{2\chi-1}{4\chi^2+9} \Rightarrow$$

$$\int d\left(\frac{1}{\chi}\psi\right)=\int\frac{2\chi-1}{4\chi^2+9}d\chi \Rightarrow \frac{\psi}{\chi}=\frac{1}{4}\ln\left(4\chi^2+9\right)-\frac{1}{6}\tau o\xi\varepsilon\phi\left(\frac{2\chi}{3}\right)+c \Rightarrow$$

$$\boxed{\psi=\frac{\chi}{4}\ln\left(4\chi^2+9\right)-\frac{\chi}{6}\tau o\xi\varepsilon\phi\left(\frac{2\chi}{3}\right)+c\chi} \quad , \ c \text{ σταθερά}$$

4.



$(PT)=\kappa \Rightarrow (PT)^2=\kappa^2 \Rightarrow$

$$\left(c\rho-ct\right)^2+\left(\frac{c}{\rho}-\frac{c}{t}\right)^2=\kappa^2 \Rightarrow$$

$$c^2\left(\rho-t\right)^2+c^2\frac{\left(\rho-t\right)^2}{\rho^2t^2}=\kappa^2 \Rightarrow$$

$$c^2\left(\rho-t\right)^2\left(1+\frac{1}{\rho^2t^2}\right)=\kappa^2$$

Ισχύει $\left(\rho-t\right)^2=\left(\rho+t\right)^2-4\rho t \Rightarrow$

$$c^2\left[\left(\rho+t\right)^2-4\rho t\right]\left[1+\frac{1}{(\rho t)^2}\right]=\kappa^2 \quad (1)$$

Μ μέσο του ΤΡ $\chi_M=\dfrac{c\rho+ct}{2}=\dfrac{c(\rho+t)}{2}$ *(2)*, $\quad \psi_M=\dfrac{\frac{c}{\rho}+\frac{c}{t}}{2}=\dfrac{c(\rho+t)}{2\rho t}$ *(3)*

$$\frac{(2)}{(3)} \Rightarrow \frac{\chi}{\psi}=\frac{\frac{\cancel{c(\rho+t)}}{\cancel{2}}}{\frac{\cancel{c(\rho+t)}}{\cancel{2}\rho t}} \Rightarrow \frac{\chi}{\psi}=\rho t \quad (4), \qquad (2) \Rightarrow \rho+t=\frac{2\chi}{c} \Rightarrow$$

Αντικαθιστώντας στην (1) $\Rightarrow c^2\left[\left(\dfrac{2\chi}{c}\right)^2-4\left(\dfrac{\chi}{\psi}\right)\right]\left[1+\dfrac{1}{\left(\frac{\chi}{\psi}\right)^2}\right]=\kappa^2 \Rightarrow$

$$c^2\left[\frac{4\chi^2}{c^2}-\frac{4\chi}{\psi}\right]\left[1+\frac{\psi^2}{\chi^2}\right]=\kappa^2 \Rightarrow \cancel{c^2}\,\frac{4\chi^{\cancel{2}}\psi-4\cancel{\chi}c^2}{\cancel{c^2}\psi}\cdot\frac{\chi^2+\psi^2}{\chi^{\cancel{2}}}=\kappa^2 \Rightarrow$$

$\boxed{4\left(\chi\psi - c^2\right)\left(\chi^2 + \psi^2\right) = \kappa^2 \chi\psi}$ *Εξίσωση του γεωμετρικού τόπου*

5. *ν μαθητές σε ευθεία γραμμή, 2 αδέλφια*

(α) $P_1 = 1 - P$*(τα δύο αδέλφια να στέκονται το ένα δίπλα στο άλλο)* $\Rightarrow$

$$P_1 = 1 - \frac{(\nu-1)!\cdot 2!}{\nu!} \Rightarrow P_1 = 1 - \frac{\cancel{(\nu-1)!}\cdot 2}{\cancel{(\nu-1)!}\cdot \nu} \Rightarrow P_1 = 1 - \frac{2}{\nu} \Rightarrow \boxed{P_1 = \frac{\nu-2}{\nu}}$$

(β) $P_1 > \frac{3}{4} \Rightarrow \frac{\nu-2}{\nu} > \frac{3}{4} \Rightarrow \frac{\nu-2}{\nu} - \frac{3}{4} > 0 \Rightarrow \frac{4\nu-8-3\nu}{4\nu} > 0 \Rightarrow \frac{\nu-8}{4\nu} > 0 \overset{\nu>0}{\Rightarrow}$

$\nu - 8 > 0 \Rightarrow \nu > 8 \Rightarrow \boxed{\nu_{\min} = 9}$

(γ) $\nu \leq 365$

$$P_2 = 1 - P(\text{όλοι διαφορετικοί ημερομηνία}) \Rightarrow P_2 = 1 - \frac{\Delta_\nu^{365}}{\delta_\nu^{365}} \Rightarrow P_2 = 1 - \frac{\frac{365!}{(365-\nu)!}}{365^\nu} \Rightarrow$$

$$\boxed{P_2 = 1 - \frac{365!}{(365-\nu)!\cdot 365^\nu}}$$