ΠΡΟΤΕΙΝΟΜΕΝΕΣ ΛΥΣΕΙΣ

ΜΕΡΟΣ Α΄

1. $\psi = \tau o\xi \varepsilon\varphi 3\chi \Rightarrow \dfrac{d\psi}{d\chi} = \dfrac{(3\chi)'}{1+(3\chi)^2} \Rightarrow \dfrac{d\psi}{d\chi} = \dfrac{3}{1+9\chi^2}$

2. $L = \lim\limits_{x\to 0} \dfrac{e^x \eta\mu\chi + \chi}{\sigma\upsilon\nu\chi + 2\chi - 1} = \dfrac{e^0 \cdot \eta\mu 0 + 0}{\sigma\upsilon\nu 0 + 2\cdot 0 - 1} = \left(\dfrac{0}{0}\right)$ απροσδιόριστη μορφή

$$L = \lim_{\chi\to 0} \frac{(e^x \eta\mu\chi + \chi)'}{(\sigma\upsilon\nu\chi + 2\chi - 1)'} = \lim_{\chi\to 0} \frac{e^x\eta\mu\chi + e^x\sigma\upsilon\nu\chi + 1}{-\eta\mu\chi + 2} = \frac{e^0\eta\mu 0 + e^0\sigma\upsilon\nu 0 + 1}{-\eta\mu o + 2} = \frac{2}{2} = 1$$

3. *τιμή αγοράς*   *τιμή πώλησης*

100   108

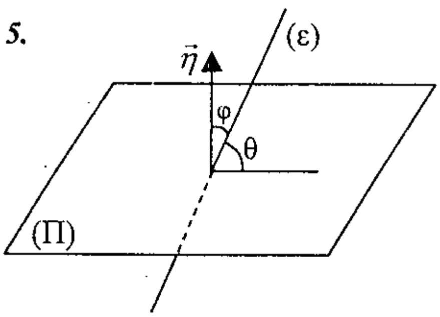
χ;   37800 $\chi = \dfrac{37800 \cdot 100}{108} \Rightarrow \chi = £35000$

Το οικόπεδο αγοράστηκε ***£35 000***

4. $\dfrac{d^2\psi}{d\chi^2} + \dfrac{d\psi}{d\psi} - 6\psi = 0$ Βοηθητική εξίσωση: $m^2+m-6=0 \Rightarrow (m+3)(m-2)=0 \Rightarrow$

$m_1 = -3$, $m_2 = 2$. Γενική Λύση: $\psi = A\cdot e^{-3\chi} + B\cdot e^{2\chi}$

5.



(ε) : $\dfrac{\chi+3}{2} = \dfrac{\psi-1}{2} = z \Rightarrow \dfrac{\chi+3}{2} = \dfrac{\psi-1}{2} = \dfrac{z}{1}$

$\Rightarrow$ (ε) // $\vec{u} = 2\vec{i} + 2\vec{j} + \vec{k}$

(Π): $5\chi+4\psi-3z+2=0 \Rightarrow$ (Π) $\perp \vec{\eta} = 5\vec{i} + 4\vec{j} - 3\vec{k}$

$$\sigma\upsilon\nu\varphi = \frac{A_1\cdot A_2 + B_1\cdot B_2 + \Gamma_1\cdot\Gamma_2}{\sqrt{A_1^2 + B_1^2 + \Gamma_1^2}\cdot\sqrt{A_2^2+B_2^2+\Gamma_2^2}}$$

$$\sigma\upsilon\nu\phi = \frac{2\cdot 5 + 2\cdot 4 + 1\cdot(-3)}{\sqrt{2^2+2^2+1^2}\cdot\sqrt{5^2+4^2+(-3)^2}} \Rightarrow \sigma\upsilon\nu\phi = \frac{\cancel{15}^{\,5}}{\cancel{3}\cdot\sqrt{50}} \Rightarrow$$

$$\sigma\upsilon\nu\phi = \frac{\cancel{5}}{\cancel{5}\cdot\sqrt{2}} \Rightarrow \sigma\upsilon\nu\phi = \frac{\sqrt{2}}{2} \Rightarrow \varphi = 45° \Rightarrow \theta = 90° - \varphi \Rightarrow \theta = 90° - 45° \Rightarrow \boldsymbol{\theta = 45°}$$

6. $A=\begin{pmatrix}2 & -1\\ 1 & 3\end{pmatrix}$ και $B=\begin{pmatrix}9 & 8\\ 1 & 11\end{pmatrix}$

(α) $A=\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}$, $A^{-1}=\dfrac{1}{|A|}\begin{pmatrix}\delta & -\beta\\ -\gamma & \alpha\end{pmatrix}$

$|A|=\begin{vmatrix}2 & -1\\ 1 & 3\end{vmatrix}=6+1=7 \Rightarrow A^{-1}=\dfrac{1}{7}\begin{pmatrix}3 & 1\\ -1 & 2\end{pmatrix}$

(β) $AX=B \Rightarrow X=A^{-1}\cdot B \Rightarrow X=\dfrac{1}{7}\begin{pmatrix}3 & 1\\ -1 & 2\end{pmatrix}\cdot\begin{pmatrix}9 & 8\\ 1 & 11\end{pmatrix} \Rightarrow$

$$X=\frac{1}{7}\begin{pmatrix}3\cdot 9+1\cdot 1 & 3\cdot 8+1\cdot 11\\ -1\cdot 9+2\cdot 1 & -1\cdot 8+2\cdot 11\end{pmatrix} \Rightarrow X=\frac{1}{7}\begin{pmatrix}28 & 35\\ -7 & 14\end{pmatrix} \Rightarrow X=\begin{pmatrix}4 & 5\\ -1 & 2\end{pmatrix}$$

7. $$\left(\chi+\frac{\alpha}{\chi}\right)^{10} \Rightarrow T_{\kappa+1}=\binom{10}{\kappa}\chi^{10-\kappa}\left(\frac{\alpha}{\chi}\right)^{\kappa}=\binom{10}{\kappa}\cdot\chi^{10-\kappa}\cdot\frac{\alpha^{\kappa}}{\chi^{\kappa}}=\binom{10}{\kappa}\cdot\alpha^{\kappa}\cdot\chi^{10-2\kappa} \Rightarrow$$

(i) $10-2\kappa=6 \Rightarrow 2\kappa=4 \Rightarrow \kappa=2 \Rightarrow T_3=\binom{10}{2}\cdot\alpha^2\cdot\chi^6 \Rightarrow T_3=45\alpha^2\chi^6$

(ii) $10-2\kappa=8 \Rightarrow 2\kappa=2 \Rightarrow \kappa=1 \Rightarrow T_2=\binom{10}{1}\cdot\alpha\cdot\chi^8 \Rightarrow T_2=10\cdot\alpha\cdot\chi^8$

$45\alpha^2=9\cdot 10\alpha \Rightarrow 45\alpha^2-90\alpha=0 \Rightarrow 45\alpha(\alpha-2)=0 \Rightarrow \underline{\alpha=2}$ ή $\alpha=0$ απορρίπτεται.

8.

χ_i	f_i	$\chi_i\cdot f_i$	$(\chi_i-\bar{\chi})^2$	$f_i(\chi_i-\bar{\chi})^2$
3	2	$3\cdot 2=6$	$(3-8)^2=25$	$2\cdot 25=50$
5	5	25	$(5-8)^2=9$	45
7	10	70	$(7-8)^2=1$	10
9	6	54	$(9-8)^2=1$	6
11	4	44	$(11-8)^2=9$	36
13	2	26	$(13-8)^2=25$	50
15	1	15	$(15-8)^2=49$	49
	$\Sigma f_i=30$	$\sum f_i\chi_i=240$		$\sum f_i(\chi_i-\bar{\chi})^2=246$

(α) $\bar{\chi}=\dfrac{\sum f_i\chi_i}{\sum f_i} \Rightarrow \bar{\chi}=\dfrac{240}{30} \Rightarrow \bar{\chi}=8$

(β) $\sigma=\sqrt{\dfrac{\sum f_i(\chi_i-\bar{\chi})^2}{\sum f_i}} \Rightarrow \sigma=\sqrt{\dfrac{246}{30}} \Rightarrow \sigma=\sqrt{8{,}2} \Rightarrow \sigma=2{,}86$

9. (α) 3, 4, 5, ... Α.Π. $\beta_1=3$, $\delta=1 \Rightarrow \beta_\kappa=\beta_1+(\kappa-1)\cdot\delta \Rightarrow \beta_\kappa=3+(\kappa-1)\cdot 1 \Rightarrow \beta_\kappa=\kappa+2$
4, 5, 6, ... Α.Π. $\gamma_1=4$, $\delta'=1 \Rightarrow \gamma_\kappa=\gamma_1+(\kappa-1)\cdot\delta' \Rightarrow \gamma_\kappa=4+(\kappa-1)\cdot 1 \Rightarrow \beta_\kappa=\kappa+3$

Άρα ο γενικός όρος είναι $\alpha_\kappa=\dfrac{1}{(\kappa+2)(\kappa+3)}$

(β) $\dfrac{1}{(\kappa+2)(\kappa+3)} \equiv \dfrac{A}{\kappa+2} + \dfrac{B}{\kappa+3} \Rightarrow 1 \equiv A(\kappa+3) + B(\kappa+2) \Rightarrow$

για $\kappa=-2 \Rightarrow A=1$, για $\kappa=-3 \Rightarrow B=-1$, επομένως: $\alpha_\kappa = \dfrac{1}{(\kappa+2)\ (\kappa+3)} = \dfrac{1}{\kappa+2} - \dfrac{1}{\kappa+3}$

$$\alpha_1 = \frac{1}{3} - \cancel{\frac{1}{4}}$$

$$\alpha_2 = \cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}$$

$$\alpha_3 = \cancel{\frac{1}{5}} - \cancel{\frac{1}{6}}$$

....

.... +

$$\alpha_{\nu-1} = \cancel{\frac{1}{\nu+1}} - \cancel{\frac{1}{\nu+2}}$$

$$\alpha_\nu = \cancel{\frac{1}{\nu+2}} - \frac{1}{\nu+3}$$

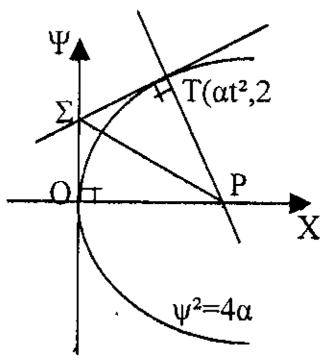
$$\sum_{\kappa=1}^{\nu} \frac{1}{(\kappa+2)\ (\kappa+3)} = \frac{1}{3} - \frac{1}{\nu+3}$$

(γ) $\displaystyle\sum_{\kappa=1}^{\infty} \frac{1}{(\kappa+2)(\kappa+3)} = \lim_{\nu\to\infty}\left(\frac{1}{3} - \frac{1}{\nu+3}\right) = \frac{1}{3}$

10. Η εξίσωση της εφαπτομένης στο Τ: $t\psi = \chi + at^2$, για $\chi=0$: $t\psi = at^2 \Rightarrow \psi = at \Rightarrow \Sigma(0, at)$
Η εξίσωση της κάθετη στο Τ: $\psi + t\chi = 2at + at^3$, για $\psi = 0$: $\chi = 2\alpha + at^2 \Rightarrow$
$P(2\alpha+at^2, 0)$, $T(at^2, 2at)$, $\Sigma(0, at)$, $P(2\alpha+at^2, 0)$

Οι συντεταγμένες του κέντρου βάρους:

$$\chi_K = \frac{\chi_1+\chi_2+\chi_3}{3}, \qquad \psi_K = \frac{\psi_1+\psi_2+\psi_3}{3}$$



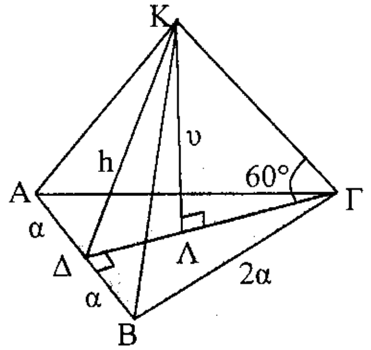
$$\chi = \frac{at^2+0+2\alpha+at^2}{3} \qquad \psi = \frac{2at+at+0}{3}$$

$$\left.\begin{array}{l} \chi = \dfrac{2\alpha+2at^2}{3} \\ \psi = at \quad \Rightarrow \quad t = \dfrac{\psi}{\alpha} \end{array}\right\} \Rightarrow 3\chi = 2\alpha + 2\alpha\cdot\frac{\psi^2}{\alpha^2} \Rightarrow$$

$$3\alpha\chi = 2\alpha^2 + 2\psi^2 \Rightarrow 2\psi^2 = 3\alpha\chi - 2\alpha^2 \Rightarrow \psi^2 = \frac{3}{2}\alpha\chi - \alpha^2 \Rightarrow \psi^2 = \frac{3\alpha}{2}\left(\chi - \frac{2}{3}\alpha\right).$$

11. Μεταξύ του 1 και του 6κ+2 υπάρχουν κ πολλαπλάσια του 6. Άρα $\dfrac{\kappa}{6\kappa+2} = \dfrac{5}{31} \Rightarrow$
$31\kappa = 30\kappa + 10 \Rightarrow \kappa = 10$. Επομένως το κουτί έχει $6\cdot 10 + 2 = $ **62 σφαίρες**

12.



$AB\Gamma$ ισόπλευρο τρίγωνο $AB=B\Gamma=\Gamma A=2\alpha$, $K\hat{\Gamma}\Delta=60°$

Δ μέσο $AB \Rightarrow A\Delta=\Delta B=\alpha$, $\Gamma\Delta$ διάμεσος, ύψος $\overset{\Delta}{AB\Gamma}$

$\overset{\Delta}{B\Gamma\Delta}$ ορθογώνιο τρίγωνο $\Rightarrow(\Gamma\Delta)^2=(2\alpha)^2-\alpha^2 \Rightarrow (\Gamma\Delta)=\alpha\sqrt{3}$

Λ κέντρο τριγώνου $\overset{\Delta}{AB\Gamma} \Rightarrow (\Gamma\Lambda)=\frac{2}{3}(\Gamma\Delta) \Rightarrow (\Gamma\Lambda)=\frac{2\alpha\sqrt{3}}{3}$,

$(\Delta\Lambda)=\frac{1}{3}(\Gamma\Delta) \Rightarrow (\Delta\Lambda)=\frac{a\sqrt{3}}{3}$

$\overset{\Delta}{\Gamma K\Lambda}$ ορθογώνιο, $K\hat{\Gamma}\Lambda=60° \Rightarrow \Gamma\hat{K}\Lambda=30° \Rightarrow$

$(K\Gamma)=2(\Gamma\Lambda) \Rightarrow (K\Gamma)=\frac{4\alpha\sqrt{3}}{3}$.

$\overset{\Delta}{\Gamma K\Lambda}$ ορθογώνιο, $\Rightarrow \varepsilon\varphi 60°=\frac{K\Lambda}{\Lambda\Gamma} \Rightarrow \sqrt{3}=\frac{K\Lambda}{\frac{2\alpha\sqrt{3}}{3}} \Rightarrow K\Lambda=\frac{2\alpha\sqrt{3}}{3}\cdot\sqrt{3} \Rightarrow (K\Lambda)=2\alpha$

$\overset{\Delta}{K\Lambda\Delta}$ ορθογώνιο $\Rightarrow (K\Delta)^2=(K\Lambda)^2+(\Lambda\Delta)^2 \Rightarrow (K\Delta)^2=(2\alpha)^2+\left(\frac{a\sqrt{3}}{3}\right)^2 \Rightarrow$

$(K\Delta)^2=4\alpha^2+\frac{3a^2}{9} \Rightarrow (K\Delta)^2=\frac{39a^2}{9} \Rightarrow (K\Delta)=\frac{a\sqrt{39}}{3}$

$E_{o\lambda}=E_\Pi+E_B$, $\quad E_\Pi=\frac{\Pi_B\cdot h}{2} \Rightarrow E_\Pi=\frac{3\cdot 2a\cdot\frac{a\sqrt{39}}{3}}{2} \Rightarrow E_\Pi=a^2\sqrt{39}$,

$E_B=\frac{\chi^2\sqrt{3}}{4}$, χ πλευρά ισοπλεύρου τριγώνου $\Rightarrow E_B=\frac{4\alpha^2\sqrt{3}}{4} \Rightarrow E_B=\alpha^2\sqrt{3}$

$E_{o\lambda}=a^2\sqrt{39}+\alpha^2\sqrt{3}$

$V=\frac{1}{3}\cdot E_B\cdot\upsilon \Rightarrow V=\frac{1}{3}\cdot\alpha^2\sqrt{3}\cdot(2a) \Rightarrow V=\frac{2\alpha^3\sqrt{3}}{3}$

13. $\chi+1=2\varepsilon\varphi\theta \Rightarrow d\chi=2\,\tau\varepsilon\mu^2\theta\cdot d\theta$, $\qquad \chi^2+2\chi+5=\chi^2+2\chi+1+4=(\chi+1)^2+4$

$$\int\frac{d\chi}{\left(\chi^2+2\chi+5\right)^{\frac{3}{2}}}=\int\frac{d\chi}{\left[(\chi+1)^2+4\right]^{\frac{3}{2}}}=\int\frac{2\tau\varepsilon\mu^2\theta\,d\theta}{\left(4\varepsilon\phi^2\theta+4\right)^{\frac{3}{2}}}=\int\frac{2\tau\varepsilon\mu^2\theta\,d\theta}{\left[4\left(\varepsilon\phi^2\theta+1\right)\right]^{\frac{3}{2}}}=\int\frac{2\tau\varepsilon\mu^2\theta\,d\theta}{4^{\frac{3}{2}}\left(\tau\varepsilon\mu^2\theta\right)^{\frac{3}{2}}}$$

$$=\int\frac{2\tau\varepsilon\mu^2\theta\,d\theta}{8\tau\varepsilon\mu^3\theta}=\frac{1}{4}\int\frac{d\theta}{\tau\varepsilon\mu\theta}=\frac{1}{4}\int\sigma\upsilon\nu\theta\,d\theta=\frac{1}{4}\eta\mu\theta+c=\frac{1}{4}\frac{\varepsilon\phi\theta}{\sqrt{1+\varepsilon\phi^2\theta}}+c$$

$$=\frac{1}{4}\frac{\frac{\chi+1}{2}}{\sqrt{1+\left(\frac{\chi+1}{2}\right)^2}}+c=\frac{\chi+1}{4\sqrt{\chi^2+2\chi+5}}+c$$

14. Έστω η εξίσωση του ζητούμενου κύκλου είναι $\chi^2+\psi^2+2g\chi+2f\psi+c=0$

Ο κύκλος περνά από το σημείο $(4,2)$ και $(1,1)$

$(4,2) \Rightarrow \chi=4,\ \psi=2 \Rightarrow 4^2+2^2+2\cdot g\cdot 4+2\cdot f\cdot 2+c=0 \Rightarrow 8g+4f+c=-20 \quad (1)$

$(1,1) \Rightarrow \chi=1, \psi=1 \Rightarrow 1^2+1^2+2\cdot g\cdot 1+2\cdot f\cdot 1+c=0 \Rightarrow 2g+2f+c=-2 \Rightarrow 2(g+f)+c=-2$ (2)

Ο κύκλος $\chi^2+\psi^2+2g\chi+2f\psi+c=0$ εφάπτεται της ευθείας $\psi = \chi \Rightarrow$ η λύση του συστήματος των εξισώσεων τους έχει μοναδική λύση δηλ. $\Delta = 0$

$$\left.\begin{array}{l} \chi^2+\psi^2+2g\chi+2f\chi+\psi=0 \\ \psi=\chi \end{array}\right\} \Rightarrow \chi^2+\chi^2+2g\chi+2f\chi+c=0 \Rightarrow$$

$$2\chi^2+2(g+f)\chi+c=0 \overset{\Delta=0}{\Rightarrow} [2(g+f)]^2-4\cdot 2\cdot c=0 \Rightarrow 4(g+f)^2-4\cdot 2c=0 \Rightarrow (g+f)^2-2c=0 \quad (3)$$

Άρα έχουμε το σύστημα των εξισώσεων (1) $\wedge$ (2) $\wedge$ (3)

$$\left.\begin{array}{ll} (1) & 8g+4f+c=-20 \\ (2) & 2(g+f)+c=-2 \Rightarrow \\ & g+f=-\dfrac{c+2}{2} \\ (3) & (g+f)^2-2c=0 \end{array}\right| \overset{(2)}{\underset{(3)}{\Rightarrow}} \left(-\frac{c+2}{2}\right)^2-2c=0 \Rightarrow \frac{(c+2)^2}{4}=2c \Rightarrow$$

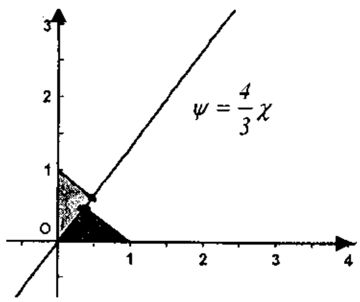
$c^2+4c+4=8c \Rightarrow c^2-4c+4=0 \Rightarrow (c-2)^2=0 \Rightarrow c=2$

$(1) \Rightarrow 8g+4f+2=-20 \Rightarrow 4g+2f=-11,\quad (2) \Rightarrow 2(g+f)+2=-2 \Rightarrow 2g+2f=-4$

$$\left.\begin{array}{l} 4g+2f=-11 \\ 2g+2g=-4 \end{array}\right\} \Rightarrow g=-\frac{7}{2},\quad f=\frac{3}{2}$$

Άρα η εξίσωση του κύκλου είναι : $\boldsymbol{\chi^2+\psi^2-7\chi+3\psi+2=0}$.

15. $T: \begin{pmatrix}1\\0\end{pmatrix} \rightarrow \begin{pmatrix}\gamma\\\delta\end{pmatrix}$ τότε $\dfrac{\delta}{\gamma-1}=-\dfrac{3}{4} \Rightarrow 4\delta=-\gamma+3$



αλλά $\delta=\dfrac{4}{3}\gamma \Rightarrow \dfrac{16}{3}\gamma=-3\gamma+3 \Rightarrow \dfrac{25}{3}\gamma=3 \Rightarrow \gamma=\dfrac{9}{25}$

$\delta=\dfrac{4}{3}\cdot\dfrac{9}{25} \Rightarrow \delta=\dfrac{12}{25}$ άρα $T: \begin{pmatrix}1\\0\end{pmatrix} \longrightarrow \begin{pmatrix}\frac{9}{25}\\\frac{12}{25}\end{pmatrix}$

$T: \begin{pmatrix}0\\1\end{pmatrix} \rightarrow \begin{pmatrix}\alpha\\\beta\end{pmatrix}$ τότε $\dfrac{\beta-1}{\alpha}=-\dfrac{3}{4} \Rightarrow 4\beta-4=-3\alpha$, άλλα $\beta=\dfrac{4}{3}\alpha \Rightarrow 4\cdot\dfrac{4}{3}\alpha-4+3\alpha=0$

$\Rightarrow 25\alpha=12 \Rightarrow \alpha=\dfrac{12}{25} \Rightarrow \beta=\dfrac{4}{3}\cdot\dfrac{12}{25} \Rightarrow \beta=\dfrac{16}{25}$, άρα $T: \begin{pmatrix}0\\1\end{pmatrix} \longrightarrow \begin{pmatrix}\frac{12}{25}\\\frac{16}{25}\end{pmatrix}$

Άρα $T=\dfrac{1}{25}\begin{pmatrix}9 & 12\\12 & 16\end{pmatrix}$

<u>Προτεινόμενες λύσεις για τις τρεις διαφορετικές ασκήσεις για το Ενιαίο Λύκειο</u>

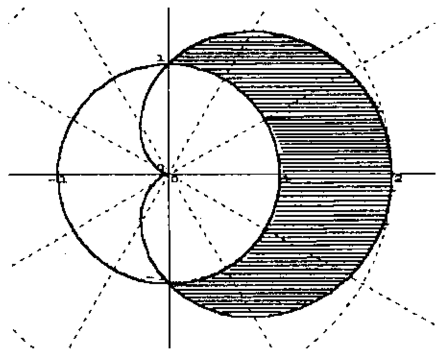
7. $f(\chi)=\chi^3-\chi-1 \quad \chi\in\mathbf{R}$, συνεχής συνάρτηση $\forall\chi\in\mathbf{R}$, $\Rightarrow f'(\chi)=3\chi^2-1$

$$\left.\begin{array}{l} f(1)=1^3-1-1=-1<0 \\ f(2)=2^3-2-1=5>0 \end{array}\right\} \Rightarrow f(1)\cdot f(2)<0 \Rightarrow$$ υπάρχει μία τουλάχιστον ρίζα στο διάστημα $[1,2]$

$$\chi_1 = \chi_0 - \frac{f(\chi_0)}{f'(\chi_0)} = 1,2 - \frac{f(1,2)}{f'(1,2)} = 1,2 - \frac{-0,472}{3,32} = 1,2 + 1,1422 = 1,3422 \quad \simeq \quad 1,342$$

9. $|z - 1 - 2i| = |z - 4 - i| \Rightarrow |\chi + \psi i - 1 - 2i| = |\chi + \psi i - 4 + i| \Rightarrow$
$|\chi - 1 + (\psi - 2)i| = |\chi - 4 + (\psi + 1)i| \Rightarrow (\chi - 1)^2 + (\psi - 2)^2 = (\chi - 4)^2 + (\psi + 1)^2 \Rightarrow$
$\chi^2 - 2\chi + 1 + \psi^2 - 4\psi + 4 = \chi^2 - 8\chi + 16 + \psi^2 + 2\psi + 1 \Rightarrow 6\chi - 6\psi = 12 \Rightarrow$ $\boldsymbol{\chi - \psi = 2}$

13. (α) $\left.\begin{matrix} \rho = 1 + \sigma\upsilon\nu\theta \\ \rho = 1 \end{matrix}\right\} \Rightarrow 1 + \sigma\upsilon\nu\theta = 1$



$$\Rightarrow \sigma\upsilon\nu\theta = 0 \Rightarrow \theta = \frac{\pi}{2}, \quad \theta = \frac{3\pi}{2}$$

Σημεία τομής: $A\left(1, \frac{\pi}{2}\right)$, $B\left(1, \frac{3\pi}{2}\right)$

(β) $E = 2 \cdot \frac{1}{2} \int_0^{\frac{\pi}{2}} (1 + \sigma\upsilon\nu\theta)^2 d\theta - \frac{\pi}{2}$

$$= \int_0^{\frac{\pi}{2}} (1 + 2\sigma\upsilon\nu\theta + \sigma\upsilon\nu^2\theta) d\theta - \frac{\pi}{2} = \int_0^{\frac{\pi}{2}} \left(1 + 2\sigma\upsilon\nu\theta + \frac{1 + \sigma\upsilon\nu 2\theta}{2}\right) d\theta - \frac{\pi}{2} =$$

$$= \left[\theta + 2\eta\mu\theta + \frac{\theta}{2} + \frac{1}{4}\eta\mu 2\theta\right]_0^{\frac{\pi}{2}} - \frac{\pi}{2} = \left[\frac{3\theta}{2} + 2\eta\mu\theta + \frac{1}{4}\eta\mu 2\theta\right]_0^{\frac{\pi}{2}} - \frac{\pi}{2} =$$

$$= \frac{3\pi}{4} + 2 - \frac{\pi}{2} = \frac{\pi}{4} + 2 = \frac{\pi + 8}{4}$$

ΜΕΡΟΣ Β΄

1. α) $\psi = (\chi - 2) \cdot e^{\chi}$ π.ο. $\chi \in \mathbf{R}$

για $\chi = 0 \Rightarrow \psi = -2$ άρα τέμνει τον άξονα των ψ στο $(0, -2)$

για $\psi = 0 \Rightarrow \chi = 2$ άρα τέμνει τον άξονα των χ στο $(2, 0)$

$$\frac{d\psi}{d\chi} = e^{\chi} + (\chi - 2)e^{\chi} = (\chi - 1)e^{\chi}$$

$$\frac{d\psi}{d\chi} = 0 \Rightarrow (\chi - 1)e^{\chi} = 0 \Rightarrow \chi = 1, \quad (e^{\chi} \neq 0)$$

χ		1	
$\frac{d\psi}{d\chi}$	$-$	0	$+$
ψ	↘	min $(1, -e)$	↗

$$\frac{d^2\psi}{d\chi^2} = e^{\chi} + (\chi - 1)e^{\chi} = \chi e^{\chi}$$

$$\frac{d^2\psi}{d\chi^2} = 0 \Rightarrow \chi e^{\chi} = 0 \Rightarrow \chi = 0, \quad (e^{\chi} \neq 0)$$

χ		0	
$\frac{d^2\psi}{d\chi^2}$	$-$	0	$+$
ψ	$\cap$	σ.κ. $(0, -2)$	$\cup$

$$\lim_{\chi \to -\infty} [(\chi - 2)e^{\chi}] = (-\infty \cdot 0) = \lim_{\chi \to -\infty} \frac{\chi - 2}{e^{-\chi}} = \left(\frac{-\infty}{\infty}\right) = \lim_{\chi \to -\infty} \frac{(\chi - 2)'}{(e^{-\chi})'} = \lim_{\chi \to -\infty} \frac{1}{-e^{-\chi}} = \frac{1}{-\infty} = 0^-$$

άρα η ευθεία $\psi = 0$ δηλαδή ο άξονας των χ είναι Ο.Α. στην περιοχή του $-\infty$

$\lim_{\chi\to\infty}\left[\,(\chi-2)\ e^{\chi}\,\right]=(+\infty)\cdot(+\infty)=+\infty$ *άρα δεν υπάρχει*

Ο.Α. στην περιοχή του $+\infty$

(β) Σημείο τομής των καμπυλών

$$\left.\begin{array}{l}\psi=(\chi-2)\cdot e^{\chi}\\ \psi=e^{\chi}\end{array}\right\}\Rightarrow (\chi-2)\cdot e^{\chi}=e^{\chi}\Rightarrow e^{\chi}\cdot(\chi-3)=0\Rightarrow$$

$\chi=3,\ \psi=e^3 \Rightarrow$ *Σημείο τομής* $(3,e^3)$

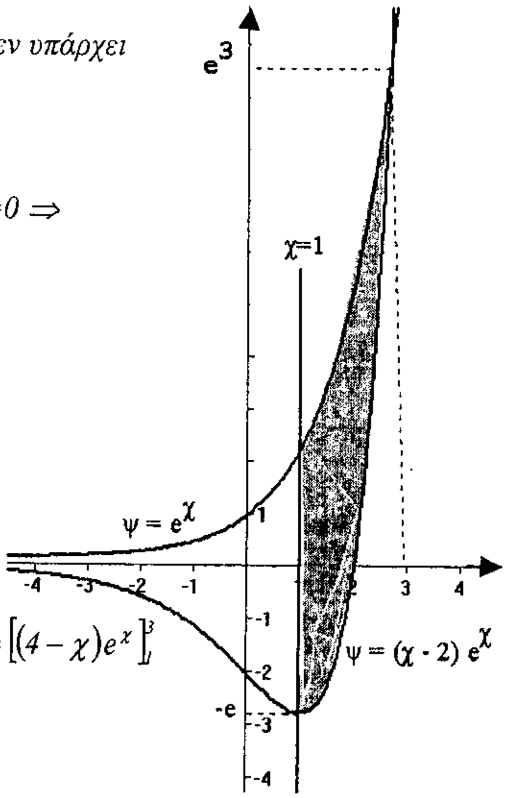
$$E=\int_1^3\left[e^{\chi}-(\chi-2)e^{\chi}\right]d\chi=\int_1^3(1-\chi+2)e^{\chi}\,d\chi$$

$$=\int_1^3(3-\chi)e^{\chi}\,d\chi=\int_1^3(3-\chi)d\left(e^{\chi}\right)=$$

$$\left[(3-\chi)e^{\chi}\right]_1^3-\int_1^3 e^{\chi}\,d(3-\chi)=$$

$$\left[(3-\chi)e^{\chi}\right]_1^3-\int_1^3 e^{\chi}\,d\chi\quad=\left[(3-\chi)e^{\chi}+e^{\chi}\right]_1^3=\left[(4-\chi)e^{\chi}\right]_1^3$$

$$=e^3-3e$$



2. $\dfrac{1}{\psi^5}\cdot\dfrac{d\psi}{d\chi}-\dfrac{2}{2\psi^4\chi}=5\chi^2\quad (i)$

(α) $\omega=\dfrac{1}{\psi^4}\Rightarrow\dfrac{d\omega}{d\chi}=\dfrac{d\omega}{d\psi}\cdot\dfrac{d\psi}{d\chi}\Rightarrow\dfrac{d\omega}{d\chi}=-\dfrac{4}{\psi^5}\cdot\dfrac{d\psi}{d\chi}\Rightarrow\dfrac{1}{\psi^5}\cdot\dfrac{d\psi}{d\chi}=-\dfrac{1}{4}\cdot\dfrac{d\omega}{d\chi}$

Αντικαθιστώ στην (i) και έχω: $-\dfrac{1}{4}\cdot\dfrac{d\omega}{d\chi}-\dfrac{1}{2\chi}\omega=5\chi^2\Rightarrow\dfrac{d\omega}{d\chi}+\dfrac{2}{\chi}\omega=-20\chi^2$

$$\mathrm{I}=e^{\int\frac{2}{\chi}d\chi}=e^{2\ln\chi}=e^{\ln\chi^2}=\chi^2$$

Άρα $\chi^2\cdot\dfrac{d\omega}{d\chi}+2\chi\omega=-20\chi^4\Rightarrow\dfrac{d}{d\chi}\left(\chi^2\omega\right)=-20\chi^4\Rightarrow\chi^2\omega=-4\chi^5+c\Rightarrow$

$\omega=-4\chi^3+\dfrac{c}{\chi^2},\quad\dfrac{1}{\psi^4}=-4\chi^3+\dfrac{c}{\chi^2}\Rightarrow\boxed{\psi^4=\dfrac{\chi^2}{c-4\chi^5}}$

(β) $\psi=\dfrac{1}{2}\Rightarrow$ *όταν* $\chi=1\Rightarrow\dfrac{1}{c-4}=\dfrac{1}{16}\Rightarrow c-4=16\Rightarrow c=20$

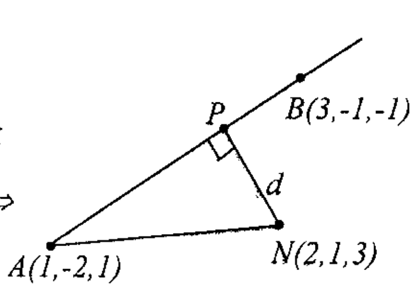
Άρα η ειδική λύση της (i) είναι: $\boxed{\psi^4=\dfrac{\chi^2}{20-4\chi^5}}$

3. (α) $A(1,-2,1)$ $\quad B(3,-1,-1)$ $\quad N(2,1,3)$

$\overrightarrow{OA}=\vec{i}-2\vec{j}+\vec{k}$, $\overrightarrow{OB}=3\vec{i}-\vec{j}-\vec{k}$, $\overrightarrow{ON}=2\vec{i}+\vec{j}+3\vec{k}$

$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}\Rightarrow\overrightarrow{AB}=\left(3\vec{i}-\vec{j}-\vec{k}\right)-\left(\vec{i}-2\vec{j}+\vec{k}\right)\Rightarrow$

$\overrightarrow{AB}=2\vec{i}+\vec{j}-2\vec{k}$



Άρα η διανυσματική εξίσωση της ευθείας (ε) είναι: $\vec{r} = \overrightarrow{OA} + \lambda\overrightarrow{AB} \Rightarrow$

(ε) $\vec{r} = \vec{i} - 2\vec{j} + \vec{k} + \lambda\left(2\vec{i} + \vec{j} - 2\vec{k}\right)$

(β) $\overrightarrow{AN} = \overrightarrow{ON} - \overrightarrow{OA} = (2\vec{i} + \vec{j} + 3\vec{k}) - (\vec{i} - 2\vec{j} + \vec{k}) = \vec{i} + 3\vec{j} + 2\vec{k} \Rightarrow \boxed{\overrightarrow{AN} = \vec{i} + 3\vec{j} + 2\vec{k}}$

$$(\varepsilon) \parallel \vec{\beta} = \overrightarrow{AB} = 2\vec{i} + \vec{j} - 2\vec{k} \Rightarrow \quad \overrightarrow{AN} \times \vec{\beta} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & 2 \\ 2 & 1 & -2 \end{vmatrix} = -8\vec{i} + 6\vec{j} - 5\vec{k}$$

$$d = \frac{\left|\overrightarrow{AN} \times \vec{\beta}\right|}{\left|\vec{\beta}\right|} = \frac{\sqrt{64+36+25}}{3} = \frac{\sqrt{125}}{3} = \frac{5\sqrt{5}}{3}$$

(γ) (ε): $\vec{r} = \vec{i} - 2\vec{j} + \vec{k} + \lambda\left(2\vec{i} + \vec{j} - 2\vec{k}\right) \Rightarrow \vec{r} = (1+2\lambda)\vec{i} + (-2+\lambda)\vec{j} + (1-2\lambda)\vec{k}$

(ζ): $\vec{r} = 2\vec{i} + 3\vec{j} - 2\vec{k} + \mu\left(-5\vec{i} + 2\vec{j} + 3\vec{k}\right) \Rightarrow \vec{r} = (2-5\mu)\vec{i} + (3+2\mu)\vec{j} + (-2+3\mu)\vec{k}$

$$\Rightarrow \left.\begin{array}{l} 1+2\lambda = 2-5\mu \\ -2+\lambda = 3+2\mu \\ 1-2\lambda = -2+3\mu \end{array}\right\} \Rightarrow \begin{array}{ll} 2\lambda + 5\mu = 1 & (1) \\ \lambda - 2\mu = 5 & (2) \\ -2\lambda - 3\mu = -3 & (3) \end{array} \Rightarrow \quad \begin{array}{l} (1)+(3) \Rightarrow 2\mu = -2 \Rightarrow \mu = -1 \\ \Rightarrow 2\lambda = 6 \Rightarrow \lambda = 3 \end{array}$$

Οι τιμές αυτές επαληθεύουν την (2) Άρα το σύστημα είναι συμβιβαστό και οι δύο ευθείες τέμνονται σε σημείο Η.

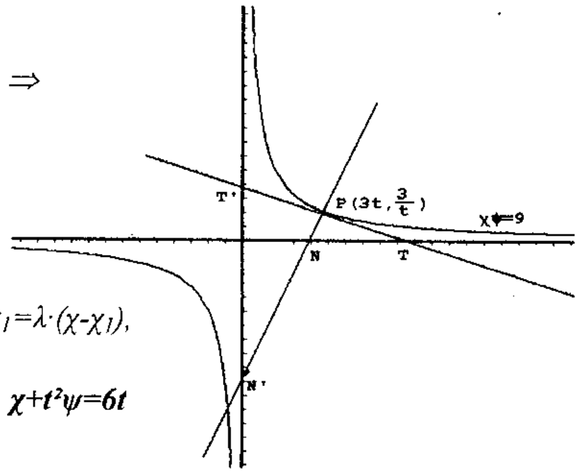
$\lambda = 3 \Rightarrow \vec{r}_H = (1+2\cdot 3)\vec{i} + (-2+3)\vec{j} + (1-2\cdot 3)\vec{k} \Rightarrow \vec{r}_H = 7\vec{i} + \vec{j} - 5\vec{k} \Rightarrow$ **Η(7,1,-5)**

(δ) ευθεία (ε): $\vec{r} = \vec{i} - 2\vec{j} + \vec{k} + \lambda\left(2\vec{i} + \vec{j} - 2\vec{k}\right)$, $(\varepsilon) \parallel (\Pi) \Rightarrow (\Pi) \parallel \vec{u} = 2\vec{i} + \vec{j} - 2\vec{k}$

ευθεία (θ): $\chi=\psi=z \Rightarrow \dfrac{\chi}{1} = \dfrac{\psi}{1} = \dfrac{z}{1}$ $(\theta) \in (\Pi) \Rightarrow (\Pi) \parallel \vec{v} = \vec{i} + \vec{j} + \vec{k}$ και περνά από το σημείο $O(0,0,0) \in (\theta) \in (\Pi)$. Άρα η καρτεσιανή εξίσωση του επιπέδου είναι :

$$\begin{vmatrix} \chi & \psi & z \\ 1 & 1 & 1 \\ 2 & 1 & -2 \end{vmatrix} = 0 \Rightarrow -3\chi + 4\psi - z = 0 \Rightarrow \mathbf{3\chi - 4\psi + z = 0}$$

4. (α) $\chi\psi=9 \Rightarrow \psi + \chi\cdot\dfrac{d\psi}{d\chi} = 0 \Rightarrow \dfrac{d\psi}{d\chi} = -\dfrac{\psi}{\chi} \Rightarrow$



$$\lambda_{\varepsilon\varphi} = -\frac{\psi}{\chi}\Bigg|_{\substack{\chi=3t \\ \psi=\frac{3}{t}}} \Rightarrow \lambda_{\varepsilon\varphi} = -\frac{\frac{3}{t}}{3t} \Rightarrow \lambda_{\varepsilon\varphi} = -\frac{1}{t^2}$$

Εφαπτομένη : $P\left(3t, \dfrac{3}{t}\right)$, $\lambda_{\varepsilon\varphi} = -\dfrac{1}{t^2}$, $\psi - \psi_1 = \lambda\cdot(\chi - \chi_1)$,

$\Rightarrow \psi - \dfrac{3}{t} = -\dfrac{1}{t^2}\cdot(\chi - 3t) \Rightarrow t^2\psi - 3t = -\chi + 3t \Rightarrow \boldsymbol{\chi + t^2\psi = 6t}$

(β) Κάθετη: $P\left(3t, \dfrac{3}{t}\right)$, $\lambda_{\varepsilon\varphi} = -\dfrac{1}{t^2} \Rightarrow \lambda_{\kappa\alpha\theta} = t^2 \Rightarrow$

$\psi - \dfrac{3}{t} = t^2\cdot(\chi - 3t) \Rightarrow t\psi - 3 = t^3\chi - 3t^4 \Rightarrow \boldsymbol{t\psi = t^3\chi - 3t^4 + 3}$

(γ) εφ/νη: $\chi + t^2\psi = 6t$ για $\psi = 0 \Rightarrow \chi = 6t \Rightarrow T(6t, 0)$, $\chi = 0 \Rightarrow \psi = \dfrac{6}{t} \Rightarrow T'\left(0, \dfrac{6}{t}\right)$

κάθετη: $t\psi = t^3\chi - 3t^4 + 3$,

για $\psi=0 \Rightarrow \chi = 3t - \frac{3}{t^3} \Rightarrow N(3t - \frac{3}{t^3}, 0)$, $\quad \chi=0 \Rightarrow \psi = \frac{3}{t} - 3t^3 \Rightarrow N'(0, \frac{3}{t} - 3t^3)$

$$(NT) = |\chi_T - \chi_N| = |6t - 3t + \frac{3}{t^3}| = |3t + \frac{3}{t^3}|$$

$$(N'T') = |\psi_{T'} - \psi_{N'}| = |\frac{6}{t} - \frac{3}{t} + 3t^3| = |\frac{3}{t} + 3t^3|$$

$$E = \frac{1}{2}\cdot|NT|\cdot|\psi_P| \Rightarrow E = \frac{1}{2}\cdot|3t + \frac{3}{t^3}|\cdot|\frac{3}{t}| \Rightarrow E = \frac{1}{2}|9 + \frac{9}{t^4}| \Rightarrow E = \frac{9(t^4+1)}{2t^4}$$

$$E' = \frac{1}{2}\cdot|N'T'|\cdot|\chi_P| \Rightarrow E' = \frac{1}{2}\cdot|\frac{3}{t} + 3t^3|\cdot|3t| \Rightarrow E' = \frac{1}{2}\cdot|9 + 9t^4| \Rightarrow E' = \frac{9(t^4+1)}{2}$$

$$\frac{1}{E} + \frac{1}{E'} = \frac{2t^4}{9(t^4+1)} + \frac{2}{9(t^4+1)} = \frac{2\cancel{(t^4+1)}}{9\cancel{(t^4+1)}} = \frac{2}{9} \Rightarrow \frac{1}{E} + \frac{1}{E'} = \frac{2}{9}$$

5. $P(A) = \frac{\Delta_6^{10}}{\delta_6^{10}} = \frac{\frac{10!}{4!}}{10^6} = \frac{151200}{1000000} = \frac{189}{1250}$, $\quad P(B) = \frac{10}{10^6} = \frac{1}{10^5} = \frac{1}{100000}$

$$P(\Gamma) = \frac{\binom{10}{2}\cdot(\delta_6^2 - 2)}{10^6} = \frac{45\cdot 62}{10^6} = \frac{2790}{1000000} = \frac{279}{100000}$$

(-2 όλοι στον ένα ή όλοι στον άλλο)

6. $f(\chi) = \begin{vmatrix} 2 & 1 & 0 \\ \chi & 4 & \chi \\ 0 & 1 & \chi \end{vmatrix} = 2\cdot(4\chi - \chi) - 1\cdot\chi^2 = 6\chi - \chi^2$

$$\int\sqrt{f(x)}\,d\chi = \int\sqrt{6\chi - \chi^2}\,d\chi = \int\sqrt{9 - (\chi-3)^2}\,d\chi$$

$$= \int\sqrt{9 - 9\eta\mu^2\theta}\cdot 3\sigma\upsilon\nu\theta\,d\theta = 9\int\sigma\upsilon\nu^2\theta\,d\theta =$$

$$= 9\int\frac{1 + \sigma\upsilon\nu 2\theta}{2}d\theta = \frac{9}{2}\int(1 + \sigma\upsilon\nu 2\theta)d\theta$$

$$= \frac{9}{2}\left(\theta + \frac{\eta\mu 2\theta}{2}\right) + c = \frac{9}{2}\cdot\theta + \frac{9}{2}\cdot\eta\mu\theta\cdot\sigma\upsilon\nu\theta + c$$

$$= \frac{9}{2}\tau o\xi\eta\mu\frac{\chi-3}{3} + \frac{9}{2}\cdot\frac{\chi-3}{3}\cdot\sqrt{1 - \frac{(\chi-3)^2}{9}} + c$$

$$= \frac{9}{2}\cdot\tau o\xi\eta\mu\frac{\chi-3}{3} + \frac{\chi-3}{2}\sqrt{6\chi - \chi^2} + c$$

$6\chi - \chi^2 = -(\chi^2 - 6\chi + 9) + 9$

$= 9 - (\chi-3)^2$

Θέτω $\chi - 3 = 3\eta\mu\theta \Rightarrow$

$d\chi = \sigma\upsilon\nu\theta\, d\theta$

$\eta\mu\theta = \frac{\chi-3}{3}$,

$\sigma\upsilon\nu\theta = \sqrt{1 - \eta\mu^2\theta}$