<u>ΠΡΟΤΕΙΝΟΜΕΝΕΣ ΛΥΣΕΙΣ</u>

1. (α) $y = x^3 - \sqrt{x} \Rightarrow \dfrac{dy}{dx} = 3x^2 - \dfrac{1}{2\sqrt{x}}$ (β) $y = x^2 \cdot e^{3x} \Rightarrow \dfrac{dy}{dx} = 2xe^{3x} + 3x^2 e^{3x}$

2.

<u>2000</u>	<u>2001</u>
100	108
x	133920

$$x = \frac{133920 \cdot 100}{108} = 124000$$

3. $L = \lim\limits_{x \to 0} \dfrac{\eta\mu 5\chi - \varepsilon\phi 3\chi}{\eta\mu 3\chi - \varepsilon\phi 2\chi}$ $\left(\dfrac{0}{0}\right)$ απροσδιοριστία

$$L = \frac{(\eta\mu 5\chi - \varepsilon\phi 3\chi)'}{(\eta\mu 3\chi - \varepsilon\phi 2\chi)'} = \frac{5\sigma\upsilon\nu 5\chi - 3\tau\varepsilon\mu^2 3\chi}{3\sigma\upsilon\nu 3\chi - 2\tau\varepsilon\mu^2\chi} = 2$$

4. (α) $\dfrac{7!}{2!} = 2520$. (β) Αρχίζουμε Α και τελειώνουν με Η: $5! = 120$

5. $\left(2x^3+\frac{a}{x}\right)^8$, $T_{\kappa+1}=\binom{8}{\kappa}\left(2\chi^3\right)^{8-\kappa}\left(\frac{\alpha}{\chi}\right)^{\kappa}=\binom{8}{\kappa}\cdot 2^{8-\kappa}\cdot\alpha^{\kappa}\cdot\chi^{24-4\kappa}$, $24-4\kappa=0\Leftrightarrow \underline{\kappa=6}$

$T_7=\binom{8}{6}\cdot 2^2\cdot a^6=112a^6 \Rightarrow 112a^6=7168\Leftrightarrow a^6=64\Leftrightarrow \boxed{a=\pm 2}$

6. (α) Εξίσωση κύκλου $x^2+y^2+2gx+2fy+c=0$

Α(2,5) : $4+25+4g+10f+c=0\Leftrightarrow \underline{4g+10f+c=-29}$

Β(0,1) : $1+2f+c=0\Leftrightarrow \underline{2f+c=-1}$, Γ0,4) : $16+8f+c=0\Leftrightarrow \underline{8f+c=-16}$

$2f+c=-1$
$\underline{8f+c=-16}\ (-)$
$-6f=15\Rightarrow f=-\frac{5}{2},\ c=4$

$4g+10\left(-\frac{5}{2}\right)+4=-29\Leftrightarrow 4g-25+4=-29\Leftrightarrow$
$4g-21=-29\Leftrightarrow g=-2$

Άρα η εξίσωση του κύκλου είναι: $\underline{x^2+y^2-4x-5y+4=0}$

(β) για $y=0$: $x^2-4x+4=0$, $\Delta=16-16=0$ άρα ο κύκλος εφάπτεται του άξονα των χ.

7. $P(A)=\frac{3}{4},\ P(A\cup B)=\frac{9}{10},\ \kappa\alpha\iota\ P(A\cap B)=\frac{9}{20}$

(α) $P(A\cup B)=P(A)+P(B)-P(A\cap B)\Rightarrow \frac{9}{10}=\frac{3}{4}+P(B)-\frac{9}{20}\Rightarrow \boxed{P(B)=\frac{3}{5}}$

$P(B-A)=P(B)-P(A\cap B)\Rightarrow P(B-A)=\frac{3}{5}-\frac{9}{20}\Rightarrow \boxed{P(B-A)=\frac{3}{20}}$

$P(A/B)=\frac{P(A\cap B)}{P(B)}\Rightarrow P(A/B)=\frac{\frac{9}{20}}{\frac{12}{20}}\Rightarrow \boxed{P(A/B)=\frac{3}{4}}$

(β) $P(A)\cdot P(B)=\frac{3}{4}\cdot\frac{3}{5}=\frac{9}{20}=P(A\cap B)$ άρα τα ενδεχόμενα Α και Β είναι ανεξάρτητα.

8. Θέτω $\tau o\xi\varepsilon\varphi\frac{1}{2}=\alpha\Rightarrow \varepsilon\varphi\alpha=\frac{1}{2}$, $0<\alpha<\frac{\pi}{4}$, $\tau o\xi\varepsilon\varphi\frac{24}{7}=\beta\Rightarrow \varepsilon\varphi\beta=\frac{24}{7}$, $0<\beta<\frac{\pi}{2}$

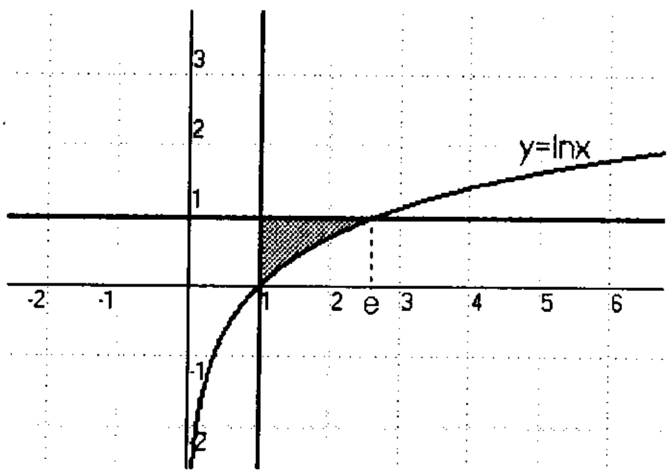
$4\tau o\xi\varepsilon\varphi\frac{1}{2}+\tau o\xi\varepsilon\varphi\frac{24}{7}=\pi$, άρα θέλω να δείξω ότι $4a+\beta=\pi$

$$\varepsilon\phi 2\alpha=\frac{2\varepsilon\phi\alpha}{1-\varepsilon\phi^2\alpha}=\frac{2\cdot\frac{1}{2}}{1-\frac{1}{4}}=\frac{4}{3},\quad \varepsilon\phi 4\alpha=\frac{2\varepsilon\phi 2\alpha}{1-\varepsilon\phi^2 2\alpha}=\frac{2\cdot\frac{4}{3}}{1-\frac{16}{9}}=-\frac{24}{7}$$

$$\varepsilon\phi\left(4\alpha+\beta\right)=\frac{\varepsilon\phi 4\alpha+\varepsilon\phi\beta}{1-\varepsilon\phi 4\alpha\cdot\varepsilon\phi\beta}=\frac{-\frac{24}{7}+\frac{24}{7}}{1+\frac{24}{7}\cdot\frac{24}{7}}=0\quad(1)\qquad \left.\begin{array}{l}0<4a<\pi\\ 0<\beta<\frac{\pi}{2}\end{array}\right\}\Rightarrow 0<4a+\beta<\frac{3\pi}{2}\quad(2)$$

Από (1) και (2) : $4a+\beta=\pi$ δηλαδή $4\tau o\xi\varepsilon\phi\frac{1}{2}+\tau o\xi\varepsilon\phi\frac{24}{7}=\pi$

9.



$$V=\pi\int_1^e\left(1-\ln^2 x\right)dx=\pi\left[x\right]_1^e-\pi\int_1^e\ln^2 x\,dx$$

$$\begin{aligned}\int\ln^2 x\,dx&=x\ln^2 x-\int x\cdot 2\ln x\cdot\frac{1}{x}dx\\ &=x\ln^2 x-2\int\ln x\,dx=\\ &=x\ln^2 x-2x\ln x+2\int x\cdot\frac{1}{x}dx\\ &=x\ln^2 x-2x\ln x+2x+c\end{aligned}$$

Άρα $V=\pi\left[x-x\ln^2 x+2x\ln x-2x\right]_1^e=\pi\left[2x\ln x-x\ln^2 x-x\right]_1^e=\pi\left(2e-e-e+1\right)=\pi\ \kappa.\mu.$

10.

$$\left.\begin{array}{l}\chi+\psi+1=0\\ y^2=4x\end{array}\right\}\begin{array}{l}\Rightarrow y=-x-1\\ \left(x+1\right)^2=4x\Rightarrow x^2-2x+1=0\Rightarrow\left(x-1\right)^2=0\Rightarrow x=1,\ y=-2\Rightarrow A\left(1,-2\right)\end{array}$$

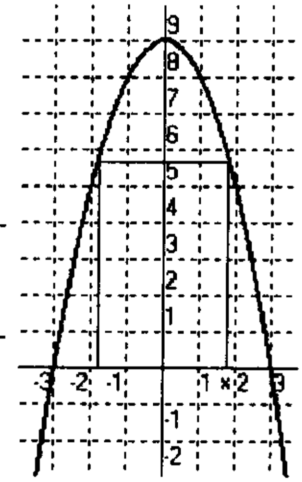
$$\left.\begin{array}{l}\psi=-x-1\\ xy=\frac{1}{4}\end{array}\right\}\Rightarrow x\left(-x-1\right)=\frac{1}{4}\Rightarrow 4x^2+4x+1=0\Rightarrow\left(2x+1\right)^2=0\Rightarrow$$

$$x=-\frac{1}{2},\ y=-\frac{1}{2}\Rightarrow B\left(-\frac{1}{2},-\frac{1}{2}\right)$$

$$\left(AB\right)^2=\left(1+\frac{1}{2}\right)^2+\left(-2+\frac{1}{2}\right)^2=\left(\frac{3}{2}\right)^2+\left(-\frac{3}{2}\right)^2=\frac{9}{2}\Rightarrow\left(AB\right)=\frac{3}{\sqrt{2}}\Rightarrow\boxed{\left(AB\right)=\frac{3\sqrt{2}}{2}}$$

11. $E = 2x\left(9 - x^2\right) = 18x - 2x^3$,

$\dfrac{dE}{dx} = 18 - 6x^2 = -6\left(x^2 - 3\right)$ $\dfrac{dE}{dx} = -6\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)$



Για $x = \sqrt{3}$ το εμβαδόν γίνεται μέγιστο και οι διαστάσεις του ορθογωνίου είναι $2\sqrt{3}$ και 6.

χ	$-\infty$		$-\sqrt{3}$		$+\sqrt{3}$		$+\infty$
$\dfrac{dy}{dx}$		$-$	0	$+$	0	$-$	
E				$\nearrow$	max	$\searrow$	

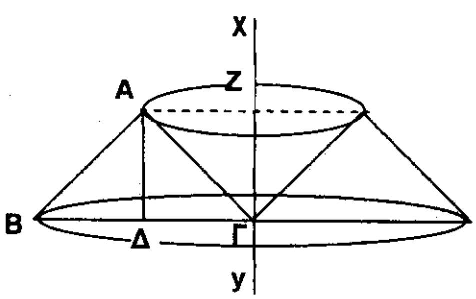
$E_{\max} = 6 \cdot 2\sqrt{3} \Rightarrow E_{\max} = 12\sqrt{3}\ \tau.\mu.$

12. $x\dfrac{dy}{dx} - 2y = x^3 + 5 \Rightarrow \dfrac{dy}{dx} - \dfrac{2}{x}y = \dfrac{x^3 + 5}{x}$, $\quad I = e^{-2\int \frac{1}{x}dx} = e^{-2\ln x} = e^{\ln \frac{1}{x^2}} = \dfrac{1}{x^2}$

$$\frac{1}{x^2}\frac{dy}{dx} - \frac{2}{x^3}y = \frac{x^3 + 5}{x^3} \Rightarrow \frac{d}{dx}\left(\frac{1}{x^2}y\right) = 1 + \frac{5}{x^3} \Rightarrow \frac{1}{x^2}y = \int\left(1 + 5x^{-3}\right)dx \Rightarrow \frac{1}{x^2}y = \frac{5x^{-2}}{-2} + c$$

$$y = x^3 + cx^2 - \frac{5}{2} \quad \text{για } x = 1,\ \ y = \frac{3}{4}:\ \ \frac{3}{4} = 1 - \frac{5}{2} + c \Rightarrow c = \frac{9}{4} \Rightarrow \boxed{y = x^3 + \frac{9}{4}x^2 - \frac{5}{2}}$$

13.



$$(\mathrm{B\Gamma})^2 = \alpha^2 + \alpha^2 = 2\alpha^2 \Rightarrow (\mathrm{B\Gamma}) = \alpha\sqrt{2}$$

$$(\mathrm{A\Delta})^2 = \alpha^2 - \left(\frac{\alpha\sqrt{2}}{2}\right)^2 \Rightarrow (\mathrm{A\Delta}) = \frac{\alpha\sqrt{2}}{2}$$

$$\mathrm{E_{B\Gamma}} = \pi(\mathrm{B\Gamma})^2 \Rightarrow \mathrm{E_{B\Gamma}} = \pi\left(\alpha\sqrt{2}\right)^2 \Rightarrow \underline{\mathrm{E_{B\Gamma}} = 2\pi\alpha^2}$$

$$\mathrm{E_{A\Gamma}} = \pi(\mathrm{AZ})(\mathrm{A\Gamma}) \Rightarrow \mathrm{E_{A\Gamma}} = \pi\frac{\alpha\sqrt{2}}{2}\cdot\alpha \Rightarrow \underline{\mathrm{E_{A\Gamma}} = \frac{\pi\alpha^2\sqrt{2}}{2}}$$

$$\mathrm{E_{AB}} = \pi(\mathrm{B\Gamma} + \mathrm{AZ})(\mathrm{AB}) \Rightarrow \mathrm{E_{AB}} = \pi\left(\alpha\sqrt{2} + \frac{\alpha\sqrt{2}}{2}\right)\cdot\alpha \Rightarrow \underline{\mathrm{E_{AB}} = \frac{3\pi\alpha^2\sqrt{2}}{2}}.$$

$$\mathrm{E}_{\sigma\tau\varepsilon\rho.} = \frac{3\pi\alpha^2\sqrt{2}}{2} + \frac{\pi\alpha^2\sqrt{2}}{2} + 2\pi\alpha^2 \Rightarrow \boxed{\mathrm{E}_{\sigma\tau\varepsilon\rho.} = 2\pi\alpha^2\left(1 + \sqrt{2}\right)cm^2}$$

14. $\dfrac{2}{2\cdot 4} + \dfrac{2}{3\cdot 5} + \dfrac{2}{4\cdot 6} + \ldots$

(α) $\beta_\nu = 2 + (\nu - 1)\cdot 1 = \nu + 1, \qquad \gamma_\nu = 4 + (\nu - 1)\cdot 1 = \nu + 3$

(β) $\dfrac{2}{(\kappa+1)(\kappa+3)} \equiv \dfrac{A}{\kappa+1}+\dfrac{B}{\kappa+3} \Rightarrow 2 \equiv A(\kappa+3)+B(\kappa+1)$

για κ = -1⇒A=1, για κ = -3⇒B=-1 άρα $a_\kappa = \dfrac{1}{\kappa+1}-\dfrac{1}{\kappa+3}$

$$\sum_{\kappa=1}^{\nu}\alpha_\kappa = \alpha_1+\alpha_2+\alpha_3+\alpha_4\ldots+\alpha_{\nu-3}+\alpha_{\nu-2}+\alpha_{\nu-1}+\alpha_\nu$$

$$\sum_{\kappa=1}^{\nu}\alpha_\kappa = \left(\frac{1}{2}-\cancel{\frac{1}{4}}\right)+\left(\frac{1}{3}-\cancel{\frac{1}{5}}\right)+\left(\cancel{\frac{1}{4}}-\cancel{\frac{1}{6}}\right)+\left(\cancel{\frac{1}{5}}-\cancel{\frac{1}{7}}\right)+\ldots+\left(\cancel{\frac{1}{\nu-2}}-\cancel{\frac{1}{\nu}}\right)+\left(\cancel{\frac{1}{\nu-1}}-\cancel{\frac{1}{\nu+1}}\right)+\left(\cancel{\frac{1}{\nu}}-\frac{1}{\nu+2}\right)+\left(\cancel{\frac{1}{\nu+1}}-\frac{1}{\nu+3}\right)$$

$$\sum_{\kappa=1}^{\nu}\alpha_\kappa = \frac{1}{2}+\frac{1}{3}-\frac{1}{\nu+1}-\frac{1}{\nu+3} \Rightarrow \sum_{\kappa=1}^{\nu}\alpha_\kappa = \frac{5}{6}-\frac{1}{\nu+2}-\frac{1}{\nu+3}$$

(γ) $\displaystyle\sum_{\kappa=1}^{\infty}\alpha_\kappa = \lim_{dx\to\infty}\left(\frac{5}{6}-\frac{1}{\nu+2}-\frac{1}{\nu+3}\right) \Rightarrow \boxed{\sum_{\kappa=1}^{\infty}\alpha_\kappa = \frac{5}{6}}$

15. (α) $\displaystyle\int_0^{\frac{\pi}{3}} x\cdot\eta\mu 3x\,dx = -\int_0^{\frac{\pi}{3}} x d\left(\frac{\sigma\upsilon\nu 3x}{3}\right) = -\frac{1}{3}\chi\,\sigma\upsilon\nu 3\chi+\frac{1}{3}\int\sigma\upsilon\nu 3\chi\,dx =$

$$= \left[-\frac{1}{3}\chi\,\sigma\upsilon\nu 3\chi+\frac{1}{9}\eta\mu 3\chi\right]_0^{\frac{\pi}{3}} = -\frac{1}{3}\cdot\frac{\pi}{3}\sigma\upsilon\nu\pi+\frac{1}{9}\eta\mu\pi-0=\frac{\pi}{9}$$

(β) $\displaystyle\int_0^2 \frac{dx}{\chi^2+2\chi+4} = \int_0^2 \frac{d(x+1)}{(\chi+1)^2+(\sqrt{3})^2} = \frac{1}{\sqrt{3}}\left[\tau o\xi\varepsilon\phi\frac{\chi+1}{\sqrt{3}}\right]_0^2 =$

$$= \frac{1}{\sqrt{3}}\left(\tau o\xi\varepsilon\phi\sqrt{3}-\tau o\xi\varepsilon\phi\frac{1}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}}\left(\frac{\pi}{3}-\frac{\pi}{6}\right) = \frac{\pi}{6\sqrt{3}}$$

<u>ΜΕΡΟΣ Β'</u>

1. $y = \dfrac{(x+1)^2}{x+2}$, π.ο. $x \in \mathbb{R}-\{-2\}$.

Για $x=0 \Rightarrow y=\dfrac{1}{2}$, άρα σημείο τομής με τον άξονα των y στο $\left(0,\dfrac{1}{2}\right)$

Για $y=0 \Rightarrow x=-1$ (διπλή), άρα σημείο τομής με τον άξονα των χ στο $(-1,0)$

$$\frac{dy}{dx} = \frac{(x+2)2(x+1)-(x+1)^2}{(x+2)^2} = \frac{(x+1)(2x+4-x-1)}{(x+2)^2} = \frac{(x+1)(x+3)}{(x+2)^2}$$

x	$-\infty$	-3		-2		-1		$+\infty$
$\frac{dy}{dx}$	+	0	−	‖	−	0	+	
y	↗	max (-3,-4)	↘	‖	↘	min (-1,0)	↗	

$x=-3 \Rightarrow y_{\max}=-4$

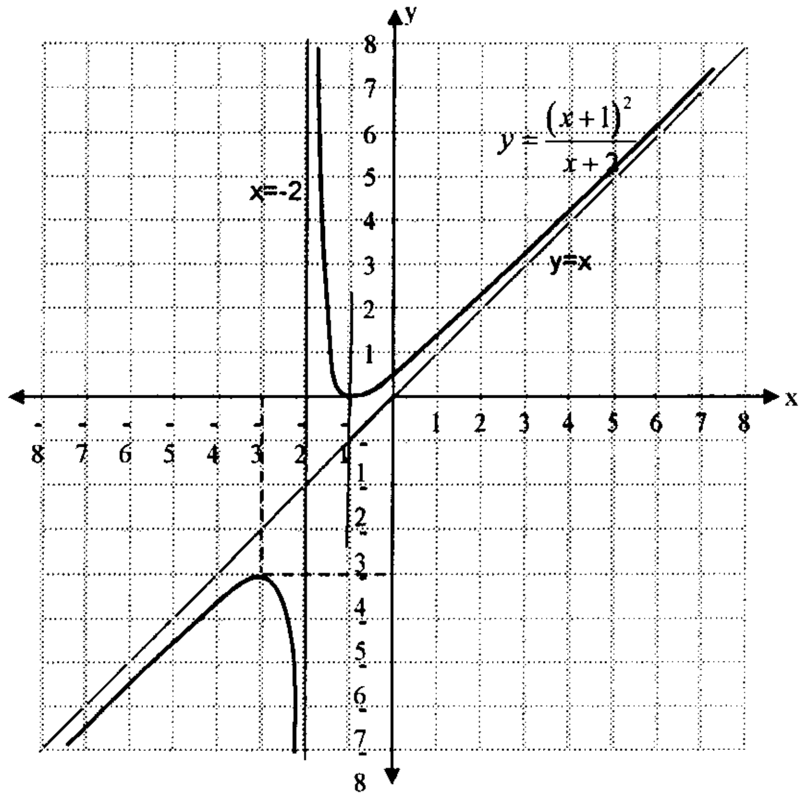
$x=-1 \Rightarrow y_{\min}=0$

$$\lim_{dx\to+\infty}\frac{x^2+2x+1}{x+2}=\lim_{dx\to+\infty}\frac{x^{\cancel{2}}\left(1+\frac{2}{x}+\frac{1}{x^2}\right)}{\cancel{x}\left(1+\frac{2}{x}\right)}=+\infty,\ \lim_{dx\to-\infty}\frac{x^2+2x+1}{x+2}=\lim_{dx\to-\infty}\frac{x^{\cancel{2}}\left(1+\frac{2}{x}+\frac{1}{x^2}\right)}{\cancel{x}\left(1+\frac{2}{x}\right)}=-\infty$$

$\lim\limits_{dx\to-2^-}\frac{(x+1)^2}{x+2}=-\infty,\ \lim\limits_{dx\to-2^+}\frac{(x+1)^2}{x+2}=+\infty \Rightarrow$ η ευθεία $x=-2$ είναι κατακόρυφη ασύμπτωτη.

$$\begin{array}{r|l} x^2+2x+1 & \underline{x+2} \\ \underline{-x^2-2x} & x \\ 1 & \end{array}$$

η ευθεία $y=x$ είναι πλάγια ασύμπτωτη.



(β) $E=\int_{-1}^{0}\left[\frac{(x+1)^2}{x+2}-x\right]dx \Rightarrow$

$E=\int_{-1}^{0}\left(\cancel{x}+\frac{1}{x+2}-\cancel{x}\right)dx \Rightarrow$

$E=\int_{-1}^{0}\frac{1}{x+2}dx \Rightarrow$

$E=\left[\ln|x+2|\right]_{-1}^{0} \Rightarrow$

$E=\ln 2-\ln 1 \Rightarrow \underline{E=\ln 2\ \tau.\mu.}$

2. $x=3\eta\mu\theta,\ -\frac{\pi}{2}<\theta<\frac{\pi}{2},\quad dx=3\sigma\upsilon\nu\theta\,d\theta$

$$\int\frac{x^2}{\sqrt{9-x^2}}dx=\int\frac{9\eta\mu^2\theta}{\sqrt{9-9\eta\mu^2\theta}}\cdot 3\sigma\upsilon\nu\theta d\theta=9\int\frac{\eta\mu^2\theta\cdot\cancel{\sigma\upsilon\nu\theta}}{\cancel{\sigma\upsilon\nu\theta}}d\theta=\frac{9}{2}\int(1-\sigma\upsilon\nu 2\theta)\,d\theta$$

$$=\frac{9}{2}\theta-\frac{9}{4}\eta\mu 2\theta+c=\frac{9}{2}\theta-\frac{9}{4}\cdot 2\eta\mu\theta\,\sigma\upsilon\nu\theta+c=\frac{9}{2}\tau o\xi\eta\mu\frac{\chi}{3}-\frac{9}{2}\cdot\frac{\chi}{3}\cdot\sqrt{1-\frac{\chi^2}{9}}+c$$

$$= \frac{9}{2} \tau o\xi\eta\mu \frac{\chi}{3} - \frac{1}{2} x\sqrt{9-x^2} + c$$

3. (α) $xy = 9 \Rightarrow y = x\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$. Στο σημείο $P\left(3p, \frac{3}{p}\right)$ $\lambda_{\varepsilon\phi} = -\frac{\frac{3}{p}}{3p} = -\frac{1}{p^2}$

Εξίσωση εφαπτομένης στο P είναι: $y - \frac{3}{p} = -\frac{1}{p^2}(x-3p) \Rightarrow p^2 y + x = 6p$

Εξίσωση εφαπτομένης στο Q είναι: $y - \frac{3}{q} = -\frac{1}{q^2}(x-3q) \Rightarrow q^2 y + x = 6q$

$$\left.\begin{aligned} p^2 y + x = 6p \\ q^2 y + x = 6q \end{aligned}\right\} \Rightarrow y = \frac{6}{p+q}, \quad x = \frac{6pq}{p+q}$$

(β) $y = \frac{6}{p+q} \Rightarrow p+q = \frac{6}{y}, \quad x = \frac{6pq}{p+q} \Rightarrow 6pq = \frac{6x}{y} \Rightarrow pq = \frac{x}{y}$

$$p^2 + q^2 = 2 \Rightarrow (p+q)^2 - 2pq = 2 \Rightarrow \frac{36}{y^2} = 2 + \frac{2x}{y} \Rightarrow \boxed{y^2 + xy = 18}$$

4. ε1: $\frac{x-1}{4} = \frac{y}{6} = \frac{z+1}{2} \Rightarrow \varepsilon_1 \parallel \vec{\alpha}(4,6,2)$, ε2: $\frac{x-3}{6} = \frac{y-3}{9} = \frac{z}{5} \Rightarrow \varepsilon_2 \parallel \vec{\beta}(6,9,5)$

$\frac{4}{6} = \frac{6}{9} \neq \frac{2}{5}$ άρα οι δύο ευθείες δεν είναι παράλληλες

$$\varepsilon_1 : \begin{aligned} \chi &= 4\lambda + 1 \\ y &= 6\lambda \\ z &= 2\lambda - 1 \end{aligned} \qquad \varepsilon_2 : \begin{aligned} \chi &= 6\mu + 3 \\ y &= 9\mu + 3 \\ z &= 5\mu \end{aligned} \qquad \left.\begin{aligned} 4\lambda + 1 &= 6\mu + 3 \\ 6\lambda &= 9\mu + 3 \\ 2\lambda - 1 &= 5\mu \end{aligned}\right\} \Rightarrow \mu = 0, \ \lambda = \frac{1}{2}$$

Το σημείο τομής των δύο ευθειών είναι $A(3,3,0)$

$\vec{a} \times \vec{\beta} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 1 \\ 6 & 9 & 5 \end{vmatrix} = 6\vec{i} - 4\vec{j} = 2(3\vec{i} - 2\vec{j})$. Το διάνυσμα $3\vec{i} - 2\vec{j}$ είναι κάθετο στο επίπεδο.

Η εξίσωση του επιπέδου είναι: $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n} \Rightarrow \vec{r} \cdot (3\vec{i} - 2\vec{j}) = (3\vec{i} + 3\vec{j}) \cdot (3\vec{i} - 2\vec{j}) \Rightarrow$

$\vec{r} \cdot (3\vec{i} - 2\vec{j}) = 9 - 6 \Rightarrow \boxed{\vec{r} \cdot (3\vec{i} - 2\vec{j}) = 3}$

5. $\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} - 3y = 5\eta\mu 2x$ (i)

(α) $\varphi(\chi) = \kappa\eta\mu 2\chi + \lambda\sigma\upsilon\nu 2\chi \Rightarrow \varphi'(\chi) = 2\kappa\sigma\upsilon\nu 2\chi - 2\lambda\eta\mu 2\chi \Rightarrow \varphi''(\chi) = -4\kappa\eta\mu 2\chi - 4\lambda\sigma\upsilon\nu 2\chi$

$$(i) \Rightarrow -4\kappa\,\eta\mu 2\chi - 4\lambda\,\sigma\upsilon\nu 2\chi + 4\kappa\,\sigma\upsilon\nu 2\chi - 4\lambda\,\eta\mu 2\chi - 3\kappa\,\eta\mu 2\chi - 3\lambda\,\sigma\upsilon\nu 2\chi = 5\eta\mu 2\chi$$

$$\Rightarrow (-7\kappa - 4\lambda)\eta\mu 2\chi + (4\kappa - 7\lambda)\sigma\upsilon\nu 2\chi = 5\eta\mu 2\chi \Rightarrow \left.\begin{array}{r} -7\kappa - 4\lambda = 5 \\ 4\kappa - 7\lambda = 0 \end{array}\right\} \Rightarrow \kappa = -\frac{7}{13},\ \lambda = -\frac{4}{13}$$

(β) $m^2 + 2m - 3 = 0 \Rightarrow (m-1)(m+3) = 0 \Rightarrow m_1 = 1,\ m_2 = -3$.

Άρα η γενική λύση είναι: $y = Ae^{x} + Be^{-3x} - \frac{7}{13}\eta\mu 2x - \frac{4}{13}\sigma\upsilon\nu 2x$

6. Έστω E_i το ενδεχόμενο στην i ρίψη να τοποθετηθεί για πρώτη φορά ένα σφαιρίδιο σε κάλπη στην οποία υπάρχει ήδη ένα άλλο σφαιρίδιο.

$$E = E_1' \cap E_2' \cap E_3' \cap \ldots \cap E_{\kappa-1}' \cap E_\kappa' \Rightarrow P(E) = P(E_1')\cdot P(E_2')\cdot P(E_3')\cdot\ldots\cdot P(E_{\kappa-1}')\cdot P(E_\kappa')$$

$$P(E) = \frac{\nu}{\nu}\cdot\frac{\nu-1}{\nu}\cdot\frac{\nu-2}{\nu}\cdot\frac{\nu-3}{\nu}\cdots\frac{\nu-\kappa+2}{\nu}\cdot\frac{\kappa-1}{\nu} \Rightarrow P(E) = \frac{\nu!(\kappa-1)}{(\nu-\kappa+1)!\,\nu^{\kappa}}$$

<u>Ασκήσεις για το 10-ωρο</u>

5. $f(x) = 3 - x - \ln x$ $\quad f(2,3) = 3 - 2,3 - 0,8329 < 0$, $\quad f(2,2) = 3 - 2,2 - 0,7884 > 0$

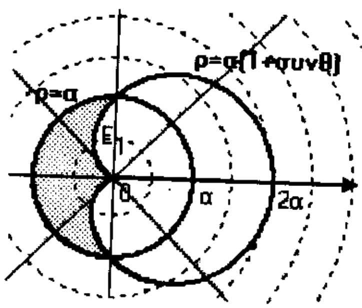
Άρα υπάρχει μια ρίζα στο διάστημα $[2,2\ ,\ 2,3]$ $\quad f'(x) = -1 - \frac{1}{x}$

$$x_1 = 2,2 - \frac{f(2,2)}{f'(2,2)} = 2,2 - \frac{0,0115}{-1-\frac{1}{2,2}} = 2,2 - \frac{0,0115}{-1,4545} = 2,2 - 0,007906 = 2,2079$$

6. $\left|\frac{z-1}{z+3i}\right| = 2 \Rightarrow \frac{|z-1|}{|z+3i|} = 2 \Rightarrow |z-1| = 2|z+3i| \Rightarrow |z-1|^2 = 4|z+3i|^2 \Rightarrow$

$$(x-1)^2 + y^2 = 4\left[x^2 + (y+3)^2\right] \Rightarrow \boxed{3x^2 + 3y^2 + 2x + 24y + 35 = 0}$$

9.



$$\left.\begin{array}{l} \rho = \alpha \\ \rho = \alpha(1+\sigma\upsilon\nu\theta) \end{array}\right\} \Rightarrow \alpha(1+\sigma\upsilon\nu\theta) = \alpha \Rightarrow 1 + \sigma\upsilon\nu\theta = 1 \Rightarrow$$

$$\left.\begin{array}{l} \sigma\upsilon\nu\theta = 0 \\ 0 \le \theta < 2\pi \end{array}\right\} \Rightarrow \begin{array}{l} \theta = \frac{\pi}{2} \Rightarrow \rho = \alpha \\ \theta = \frac{3\pi}{2} \Rightarrow \rho = \alpha \end{array}$$

Άρα τα σημεία τομής των δύο καμπυλών είναι

$A\left(\alpha, \frac{\pi}{2}\right)$ και $B\left(\alpha, \frac{3\pi}{2}\right)$

$$\beta)\ \mathrm{E}_1 = \frac{\alpha^2}{2}\int_{\frac{\pi}{2}}^{\pi}(1+\sigma\upsilon\nu\theta)^2\,d\theta = \frac{\alpha^2}{2}\int_{\frac{\pi}{2}}^{\pi}\left(1+2\sigma\upsilon\nu\theta+\sigma\upsilon\nu^2\theta\right)d\theta = \frac{\alpha^2}{2}\int_{\frac{\pi}{2}}^{\pi}\left(1+2\sigma\upsilon\nu\theta+\frac{1+\sigma\upsilon\nu 2\theta}{2}\right)d\theta$$

$$E_1 = \frac{\alpha^2}{2}\left[\theta+2\eta\mu\theta+\frac{\theta}{2}+\frac{1}{4}\eta\mu 2\theta\right]_{\frac{\pi}{2}}^{\pi} = \frac{\alpha^2}{2}\left[\frac{3\pi}{2}-2-\frac{3\pi}{4}\right] = \frac{\alpha^2}{2}\left(\frac{3\pi}{4}-2\right) = \frac{a^2(3\pi-8)}{8}$$

$$E_{\zeta\eta\tau.} = \frac{\pi\alpha^2}{2}-2\mathrm{E}_1 = \frac{\pi\alpha^2}{2}-2\frac{a^2(3\pi-8)}{8} = \frac{8\alpha^2-\pi\alpha^2}{4} = \frac{(8-\pi)\alpha^2}{4}$$