ΠΡΟΤΕΙΝΟΜΕΝΕΣ ΛΥΣΕΙΣ

ΖΗΤΗΜΑ 1°

(α) $\eta\mu(2\chi+30°) - \sigma\upsilon\nu(\chi - 12°) = 0 \Rightarrow \eta\mu(2\chi+30°) = \sigma\upsilon\nu(\chi - 12°) \Rightarrow$

$\sigma\upsilon\nu(90°-2\chi-30°) = \sigma\upsilon\nu(\chi-12°) \Rightarrow= \sigma\upsilon\nu(60°-2\chi)= \sigma\upsilon\nu(\chi-12°) \Rightarrow$

$60°-2\chi = 360°\kappa \pm (\chi - 12°) \quad \kappa \in \mathbb{Z}$

(ι) $60° - 2\chi = 360°\kappa + \chi - 12° \Rightarrow -3\chi = 360°\kappa - 72 \quad \Rightarrow \boxed{\chi = -120°\kappa + 24° \ , \ \kappa \in \mathbb{Z}}$

(ιι) $60° - 2\chi = 360°\kappa - \chi + 12° \Rightarrow -\chi = 360°\kappa + 48° \quad \Rightarrow \boxed{\chi = -360°\kappa + 48° \ , \ \kappa \in \mathbb{Z}}$

(β) $2\sigma\upsilon\nu^2\chi - 5\eta\mu\chi + 1 = 0 \Rightarrow 2(1-\eta\mu^2\chi) - 5\eta\mu\chi + 1 = 0 \Rightarrow 2 - 2\eta\mu^2\chi - 5\eta\mu\chi + 1 = 0 \Rightarrow$

$2\eta\mu^2\chi + 5\eta\mu\chi - 3 = 0 \Rightarrow (2\eta\mu\chi + 1)(\eta\mu\chi + 3) = 0 \Rightarrow$

(ι) $2\eta\mu\chi - 1 = 0 \Rightarrow \eta\mu\chi = \frac{1}{2} \Rightarrow \eta\mu\chi = \eta\mu 30° \Rightarrow$

$\chi = 360°\kappa + 30° \quad ή \quad \chi = 360°\kappa + 150°$

$\kappa = 0 \Rightarrow \boxed{\chi = 30°} \ , \ \boxed{\chi = 150°}.$

$\kappa = 1 \Rightarrow \chi > 360°$

(ιι) $\eta\mu\chi + 3 = 0 \Rightarrow \eta\mu\chi = -3$ *Αδύνατη εξίσωση*

(γ)

(ι) $$\frac{\sigma\upsilon\nu 2\beta - \sigma\upsilon\nu 2\alpha}{\eta\mu 2\alpha + \eta\mu 2\beta} = \frac{\cancel{2}\eta\mu\frac{2\beta+2\alpha}{2}\eta\mu\frac{2\alpha-2\beta}{2}}{\cancel{2}\eta\mu\frac{2\alpha+2\beta}{2}\sigma\upsilon\nu\frac{2\alpha-2\beta}{2}} = \frac{\cancel{\eta\mu(\alpha+\beta)}\eta\mu(\alpha-\beta)}{\cancel{\eta\mu(\alpha+\beta)}\sigma\upsilon\nu(\alpha-\beta)} = \varepsilon\phi(\alpha-\beta)$$

(ιι) $\sigma\upsilon\nu 4\theta\sigma\upsilon\nu 3\theta - \eta\mu 8\theta\eta\mu\theta =$

$$= \frac{1}{2}\left\{\left[\sigma\upsilon\nu(4\theta+3\theta) + \sigma\upsilon\nu(4\theta-3\theta)\right] - \left[\sigma\upsilon\nu(8\theta-\theta) - \sigma\upsilon\nu(8\theta+\theta)\right]\right\}$$

$$= \frac{1}{2}\left(\cancel{\sigma\upsilon\nu 7\theta} + \sigma\upsilon\nu\theta - \cancel{\sigma\upsilon\nu 7\theta} + \sigma\upsilon\nu 9\theta\right) = \frac{1}{2}\cdot 2\sigma\upsilon\nu\frac{9\theta+\theta}{2}\sigma\upsilon\nu\frac{9\theta-\theta}{2}$$

$$= \sigma\upsilon\nu 5\theta\,\sigma\upsilon\nu 4\theta$$

ΖΗΤΗΜΑ 2°

(α) $(\eta\mu 2\theta+\sigma\upsilon\nu 2\theta)^2 + (\eta\mu 2\theta-\sigma\upsilon\nu 2\theta)^2 - 2\sigma\upsilon\nu 2\theta =$

$$= \underline{\eta\mu^2 2\theta + \sigma\upsilon\nu^2 2\theta} + 2\cancel{\sigma\upsilon\nu 2\theta\,\eta\mu 2\theta} + \underline{\eta\mu^2 2\theta + \sigma\upsilon\nu^2 2\theta} - 2\cancel{\sigma\upsilon\nu 2\theta\,\eta\mu 2\theta} - 2\sigma\upsilon\nu 2\theta$$

$$= 1 + 1 - 2\sigma\upsilon\nu 2\theta = 2(1 - \sigma\upsilon\nu 2\theta) = 2\left(1-1+2\eta\mu^2\theta\right) = 4\eta\mu^2\theta$$

(β) $\eta\mu(\theta+60°) = 2\eta\mu\theta$ για $\theta \neq 180^0\kappa+90°$, $\kappa \in \mathbb{Z}$

$$\Rightarrow \eta\mu\theta\sigma\upsilon\nu 60° + \eta\mu 60°\sigma\upsilon\nu\theta = 2\eta\mu\theta \Rightarrow \frac{1}{2}\eta\mu\theta + \frac{\sqrt{3}}{2}\sigma\upsilon\nu\theta = 2\eta\mu\theta \Rightarrow$$

$\Rightarrow 3\eta\mu\theta = \sqrt{3}\sigma\upsilon\nu\theta$ Διαιρώ με $\sigma\upsilon\nu\theta \neq 0$ ($\theta \neq 180^0\kappa+90°$, $\kappa \in \mathbb{Z}$)

$$\Rightarrow 3\varepsilon\phi\theta = \sqrt{3} \Rightarrow \boxed{\varepsilon\phi\theta = \frac{\sqrt{3}}{3}} \Rightarrow \varepsilon\varphi\theta = \varepsilon\varphi 30° \Rightarrow \boxed{\theta = 180°\kappa + 30°} \quad \kappa \in \mathbb{Z}$$

(γ) (ι) $$\frac{1-\varepsilon\phi^2\theta}{1+\varepsilon\phi^2\theta} = \frac{1-\dfrac{\eta\mu^2\theta}{\sigma\upsilon\nu^2\theta}}{1+\dfrac{\eta\mu^2\theta}{\sigma\upsilon\nu^2\theta}} = \frac{\dfrac{\sigma\upsilon\nu^2\theta-\eta\mu^2\theta}{\cancel{\sigma\upsilon\nu^2\theta}}}{\dfrac{\cancel{\sigma\upsilon\nu^2\theta+\eta\mu^2\theta}^{\,1}}{\cancel{\sigma\upsilon\nu^2\theta}}} = \sigma\upsilon\nu^2\theta - \eta\mu^2\theta = \sigma\upsilon\nu 2\theta$$

(ιι) $$\frac{1}{\sigma\upsilon\nu 2\theta} + \varepsilon\phi 2\theta \overset{(\iota)}{=} \frac{1+\varepsilon\phi^2\theta}{1-\varepsilon\phi^2\theta} + \frac{2\varepsilon\phi\theta}{1-\varepsilon\phi^2\theta} = \frac{1+\varepsilon\phi^2\theta+2\varepsilon\phi\theta}{1-\varepsilon\phi^2\theta} = \frac{(1+\varepsilon\phi\theta)^{\cancel{2}}}{\cancel{(1+\varepsilon\phi\theta)}(1-\varepsilon\phi\theta)} =$$

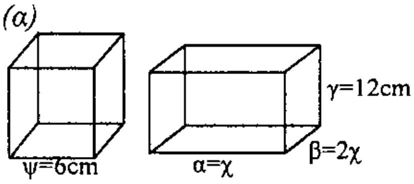
$$= \frac{1+\varepsilon\phi\theta}{1-\varepsilon\phi\theta} = \frac{1+\varepsilon\phi 45°\varepsilon\phi\theta}{1-\varepsilon\phi 45°\varepsilon\phi\theta} = \boxed{\varepsilon\phi(\theta+45°)}$$

$$\frac{1}{\sigma\upsilon\nu 2\theta} + \varepsilon\phi 2\theta = \varepsilon\phi 4\theta \overset{(\iota\iota)}{\Rightarrow} \varepsilon\phi(\theta+45°) = \varepsilon\phi 4\theta \Rightarrow$$

$$\Rightarrow \theta + 45° = 180°\kappa + 4\theta \Rightarrow 3\theta = -180°\kappa + 45° \Rightarrow \boxed{\theta = -60°\kappa + 15°}, \ \kappa \in \mathbb{Z}$$

ΖΗΤΗΜΑ 3°

(α)



(ι) $V_\kappa = \psi^3 = 6^3 = 216\,cm^3$, $\Rightarrow$ $\boxed{V_K = 216\ cm^3}$

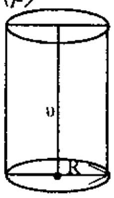
(ιι) $V_\Pi = \alpha\cdot\beta\cdot\gamma \Rightarrow 216 = \chi\cdot 2\chi\cdot 12 \Rightarrow \chi^2 = 9 \Rightarrow \chi = 3\ cm$

$\boxed{a = 3m \ , \ \beta = 6cm}$.

(ιιι) $E_K = 6\psi^2 \Rightarrow E_K = 6\cdot 6^2 \Rightarrow \boxed{E_K = 216\,cm^2}$

$$E_{\Pi\alpha\rho} = 2(\alpha\beta + \beta\gamma + \gamma\alpha) \Rightarrow E_{\Pi\alpha\rho} = 2(3\cdot 6 + 6\cdot 12 + 12\cdot 3) \Rightarrow \boxed{E_{\Pi\alpha\rho} = 252\,cm^2}$$

(β)



(ι) $R+\upsilon=9 \quad (\upsilon > R), \quad E_K=40\pi\ cm^2 \Rightarrow 2\pi R \upsilon = 40\pi \Rightarrow R\upsilon = 20$

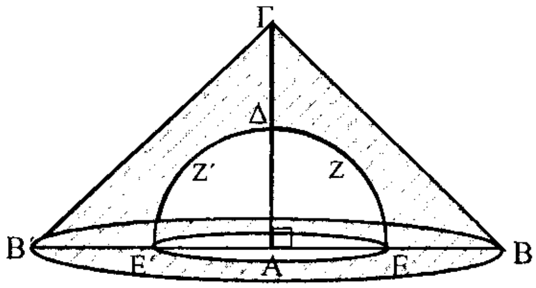
$$\left.\begin{array}{l} R+\upsilon=9 \\ R\upsilon=20 \end{array}\right. \Rightarrow \chi^2-9\chi+20=0 \Rightarrow (\chi-4)(\chi-5)=0 \Rightarrow \chi = 4 \text{ ή } \chi = 5 \quad (\upsilon>R) \Rightarrow$$

$$\boxed{\upsilon = 5cm}, \quad \boxed{R = 4cm}$$

(ιι) $E_{o\lambda} = E_K + 2\pi R^2 \Rightarrow E_{o\lambda} = 40\pi + 2\pi 4^2 \Rightarrow \boxed{E_{o\lambda} = 72\pi\ cm^2}$

$V = \pi R^2 \upsilon \Rightarrow V = \pi 4^2 \cdot 5 \Rightarrow \boxed{V = 80\pi\ cm^3}$

(γ)



$\hat{A} = 90°, \ (AB)=(A\Gamma)=4\alpha\ cm. \ (AE)=2\alpha$

$(B\Gamma)^2=(A\Gamma)^2+(AB)^2 \Rightarrow (B\Gamma)^2=16\alpha^2+16\alpha^2 \Rightarrow$

$(B\Gamma)^2=32\alpha^2 \Rightarrow B\Gamma = 4\alpha\sqrt{2}$

$V_{o\lambda} = V_{K\omega\nu o\upsilon} - V_{H\mu\iota\sigma\varphi\alpha\iota\rho\iota o\upsilon}$

$V_{o\lambda} = \frac{1}{3}\pi(AB)^2(A\Gamma) - \frac{1}{2}\frac{4}{3}\pi(AE)^3$

$V_{o\lambda} = \frac{1}{3}\pi(4\alpha)^2(4\alpha) - \frac{1}{2}\frac{4}{3}\pi(2\alpha)^3$

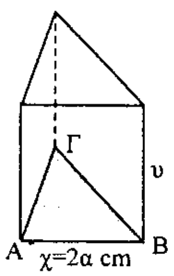
$$V_{o\lambda} = \frac{64\pi\alpha^3}{3} - \frac{16\pi\alpha^3}{3} \Rightarrow \boxed{V_{o\lambda} = 16\pi\alpha^3\ cm^3}$$

$$E_{\sigma\kappa} = E_{\kappa.\kappa\omega\nu} + E_{\eta\mu\iota\sigma\varphi} + E_{\delta\alpha\kappa\tau} \Rightarrow E_{\sigma\kappa} = \pi(AB)(B\Gamma) + \frac{1}{2}\cdot 4\pi(AE)^2 + \pi(AB)^2 - \pi(AE)^2$$

$$E_{\sigma\kappa} = \pi(AB)(B\Gamma) + \pi(AE)^2 + \pi(AB)^2 \Rightarrow E_{\sigma\kappa} = \pi(4\alpha)(4\alpha\sqrt{2}) + \pi(2\alpha)^2 + \pi(4\alpha)^2 \Rightarrow$$

$$E_{\sigma\kappa} = 16\pi\alpha^2\sqrt{2} + 4\pi\alpha^2 + 16\pi\alpha^2 \Rightarrow E_{\sigma\kappa} = 16\pi\alpha^2\sqrt{2} + 20\pi\alpha^2 \Rightarrow \boxed{E_{\sigma\kappa} = 4\pi\alpha^2\left(4\sqrt{2}+5\right)cm^2}$$

ΖΗΤΗΜΑ 4ο



(α)

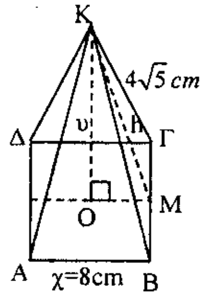
(ι) $E_{\pi\alpha\rho} = 60\alpha^2\ cm^2, \quad E_{\pi\alpha\rho} = \Pi_B \cdot \upsilon \Rightarrow 60\alpha^2 = 3\cdot 2\alpha \cdot \upsilon \Rightarrow \boxed{\upsilon = 10\alpha\ cm}$

(ιι) $E_B = \frac{\chi^2\sqrt{3}}{4} = \frac{(2\alpha)^2\sqrt{3}}{4} = \alpha^2\sqrt{3}$

$E_{o\lambda} = E_{\pi\alpha\rho} + 2E_B \Rightarrow E_{o\lambda} = 60\alpha^2 + 2\alpha^2\sqrt{3} \Rightarrow \boxed{E_{o\lambda} = 2\alpha^2\left(30+\sqrt{3}\right)\ cm^2}$

(ιιι) $V = E_B \cdot \upsilon \Rightarrow V = \alpha^2\sqrt{3}\cdot 10\alpha \Rightarrow \boxed{V = 10\alpha^3\sqrt{3}\ cm^3}$

(β)



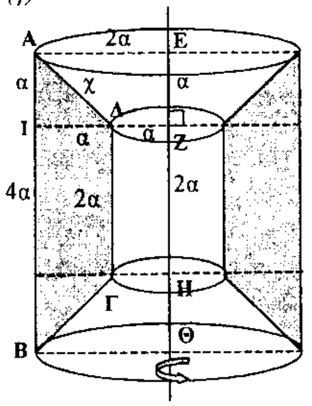
$\overset{\Delta}{KM\Gamma}: (K\Gamma)^2 = (KM)^2 + (M\Gamma)^2 \Rightarrow \left(4\sqrt{5}\right)^2 = h^2 + 4^2 \Rightarrow h^2 = 64 \Rightarrow \boxed{h = 8\ cm}$

$\overset{\Delta}{KO\Gamma}: (KM)^2 = (KO)^2 + (MO)^2 \Rightarrow (8)^2 = \upsilon^2 + 4^2 \Rightarrow \upsilon^2 = 48 \Rightarrow \boxed{\upsilon = 4\sqrt{3}\ cm}$

$E_{o\lambda} = E_\Pi + E_B = \frac{\Pi_B \cdot h}{2} + 64 = \frac{32\cdot 8}{2} + 64 = 192 \Rightarrow \boxed{E_{o\lambda} = 192 cm^2}$

$$V=\frac{E_B\cdot\upsilon}{3}=\frac{64\cdot4\cdot\sqrt{3}}{3}=\frac{256\sqrt{3}}{3}\Rightarrow \boxed{V=\frac{256\sqrt{3}}{3}cm^3}$$

(γ)



$$V_{\Sigma\chi\eta\mu.}=V_{\mu\varepsilon\gamma.\kappa\upsilon\lambda}-V_{\mu\iota\kappa\ \kappa\upsilon\lambda.}\ -2V_{K o\lambda.K\omega\nu o\upsilon}$$

$(\overset{\Delta}{\mathrm{AI\Delta}}):\ \chi^2=\alpha^2+\alpha^2\Rightarrow$

$\chi=\alpha\sqrt{2}$

$$V_{\mu\varepsilon\gamma\ \kappa\upsilon\lambda}=\pi(AE)^2\cdot AB=\pi(2\alpha)^2\cdot4\alpha=16\pi\alpha^3$$

$$V_{\mu\iota\kappa\ \kappa\upsilon\lambda}=\pi(\Delta Z)^2\cdot\Delta\Gamma=\pi(\alpha)^2\cdot2\alpha=2\pi\alpha^3$$

$$V_{K o\lambda.K\omega\nu o\upsilon}=\frac{\pi(\mathrm{EZ})}{3}\left((\mathrm{AE})^2+(\mathrm{AE})(\Delta\mathrm{Z})+(\Delta\mathrm{Z})^2\right)$$

$$=\frac{\pi\alpha}{3}\left(4\alpha^2+2\alpha^2+\alpha^2\right)=\frac{7\pi\alpha^3}{3}$$

$$V_{\Sigma\chi\eta\mu}=16\pi\alpha^3-2\pi\alpha^3-2\cdot\frac{7\pi\alpha^3}{3}=\frac{28\pi\alpha^3}{3}\Rightarrow\boxed{V_{\Sigma\chi\eta\mu.}=\frac{28\pi\alpha^3}{3}cm^3}$$

$$E_{O\lambda.}=\mathrm{E}_{\mathrm{K\,M}\varepsilon\gamma.\mathrm{K}\upsilon\lambda.}+\mathrm{E}_{\mathrm{K}\ \mu\iota\kappa.\kappa\upsilon\lambda}+2\mathrm{E}_{\mathrm{K}\ \kappa o\lambda.\kappa\omega\nu o\upsilon}$$

$$=2\pi(\mathrm{AE})(\mathrm{AB})+2\pi(\Delta\mathrm{Z})(\Delta\Gamma)+2\cdot\pi\left[(\mathrm{AE})+(\Delta\mathrm{Z})\right](\mathrm{A}\Delta)$$

$$=2\pi\cdot2\alpha\cdot4\alpha+2\pi\cdot\alpha\cdot2\alpha+2\pi(2\alpha+\alpha)\alpha\sqrt{2}$$

$$=16\pi\alpha^2+4\pi\alpha^2+6\pi\alpha^2\sqrt{2}=20\pi\alpha^2+6\pi\alpha^2\sqrt{2}\Rightarrow\boxed{E_{o\lambda}=2\pi\alpha^2\left(10+3\sqrt{2}\right)cm^2}$$

ΖΗΤΗΜΑ 5^o

(α) $\displaystyle\int_1^2\left(5\chi^4-\frac{3}{\chi^2}+2\right)dx=\int_1^2\left(5\chi^4-3\chi^{-2}+2\right)dx=\left[\chi^5+3\chi^{-1}+2\chi\right]_1^2$

$$=\left[32+\frac{3}{2}+4\right]-\left[1+3+2\right]=\frac{63}{2}$$

(β) $\psi=\dfrac{\chi^2-2\chi+1}{\chi^2+1},\quad \chi\in\mathbb{R}\Rightarrow\dfrac{dy}{dx}=\dfrac{(2\chi-2)\cdot(\chi^2+1)-(\chi^2-2\chi+1)\cdot(2\chi)}{(\chi^2+1)^2}\Rightarrow\dfrac{dy}{dx}=\dfrac{2\chi^2-2}{(\chi^2+1)^2}$

(γ) $\dfrac{dy}{dx}=0\Rightarrow2\chi^2-2=0\Rightarrow2(\chi-1)(\chi-1)=0\Rightarrow\chi=1\ ,\chi=-1$

χ	$-\infty$		-1		1		$+\infty$
$\frac{dy}{dx}$		+	0	−	0	+	
ψ		↗		↘		↗	

$x=-1\Rightarrow\psi=2\Rightarrow\max(-1,2)$

$x=1\Rightarrow\psi=0\Rightarrow\min(1,0)$

(γ) (i) $\psi=\chi^2+1,\qquad \chi=1\Rightarrow\psi=2\Rightarrow A(1,2)$,

$$\frac{dy}{dx}=2\chi\Rightarrow\lambda_{\varepsilon\phi}=2\cdot1=2\ \Rightarrow\lambda_{\kappa\alpha\theta}=-\frac{1}{2}$$

Εξίσωση εφαπτομένης στο $A(1,2)\Rightarrow\psi-2=2(\chi-1)\Rightarrow\psi=2\chi$

Εξίσωση κάθετης στο σημείο $A(1,2)\Rightarrow\psi-2=-\dfrac{1}{2}(\chi-1)\Rightarrow\chi+2\psi=5$

$$\left.\begin{array}{l} \lambda_{\varepsilon\varphi.B} = -\dfrac{1}{2} \\ B(\chi_1,\psi_1) \\ \dfrac{dy}{dx} = 2\chi \Rightarrow \lambda_{\varepsilon\varphi.B} = 2\chi_1 \end{array}\right\} \Rightarrow 2\chi_1 = -\frac{1}{2} \Rightarrow \chi_1 = -\frac{1}{4} \Rightarrow \psi_1 = \chi_1^2 + 1 = \frac{1}{16} + 1 = \frac{17}{16} \Rightarrow B\left(-\frac{1}{4}, \frac{17}{16}\right)$$

<u>ΖΗΤΗΜΑ 6ο</u>

(α) $\psi = \chi^3 + \dfrac{1}{\chi^3} \Rightarrow \underline{\dfrac{dy}{dx} = 3\chi^2 - 3\chi^{-4}} \Rightarrow \underline{\dfrac{d^2y}{dx^2} = 6\chi + 12\chi^{-5}}$

$$\chi^2 \frac{d^2y}{dx^2} + \chi\frac{dy}{dx} - 9\psi = \chi^2\left(6\chi + 12\chi^{-5}\right) + \chi\left(3\chi^2 - 3\chi^{-4}\right) - 9\left(\chi^3 + \chi^{-3}\right)$$

$$= \cancel{6\chi^3} + \cancel{12\chi^{-3}} + \cancel{3\chi^3} - \cancel{3\chi^{-3}} - \cancel{9\chi^3} - \cancel{9\chi^{-3}} = 0$$

(β) $\displaystyle\int\left(1 + \eta\mu\chi + \sigma\upsilon\nu^2\chi\right)dx = \chi - \sigma\upsilon\nu\chi + \int\frac{1 + \sigma\upsilon\nu 2\chi}{2}dx = \chi - \sigma\upsilon\nu\chi + \frac{1}{2}\left(\chi + \frac{\eta\mu 2\chi}{4}\right) + C$

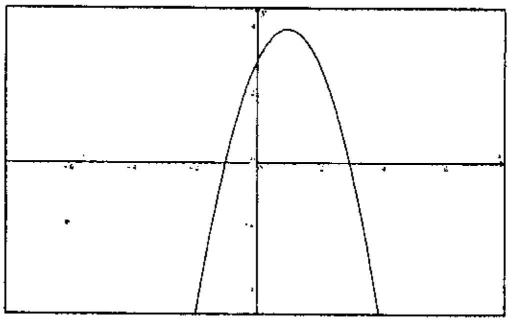
$$= \frac{3\chi}{2} - \sigma\upsilon\nu\chi + \frac{\eta\mu 2\chi}{4} + C$$

(γ) (i) $\left.\begin{array}{l} \psi = \alpha\chi^2 + \beta\chi + 3, \\ A(1,4) \Rightarrow \chi = 1, \psi = 4 \end{array}\right\} \Rightarrow 4 = \alpha + \beta + 3 \Rightarrow \alpha + \beta = 1 \quad (1)$

$$\left.\begin{array}{l} \dfrac{dy}{dx} = 2\alpha\chi + \beta \\ \dfrac{dy}{dx} = 0,\ \chi = 1 \end{array}\right\} \Rightarrow 2\alpha + \beta = o \Rightarrow \beta = -2\alpha \overset{(1)}{\Rightarrow} \alpha - 2\alpha = 1 \Rightarrow \underline{\underline{\alpha = -1}},\ \underline{\underline{\beta = 2}}$$

(ιι) $\psi = -\chi^2 + 2\chi + 3, \quad \chi = 0 \Rightarrow \psi = 3 \Rightarrow (0,3)$

$\psi = 0 \Rightarrow -\chi^2 + 2\chi + 3 = 0 \Rightarrow \chi^2 - 2\chi - 3 = 0 \Rightarrow (\chi - 3)(\chi + 1) = 0 \Rightarrow \chi = 3$ ή $\chi = -1 \Rightarrow (3,0)$, $(-1,0)$



$$E = \int_{-1}^{3}\psi\,dx = \int_{-1}^{3}\left(-\chi^2 + 2\chi + 3\right)dx$$

$$E = \left[-\frac{\chi^3}{3} + \chi^2 + 3\chi\right]_{-1}^{3} = (-9 + 9 + 9) - \left(\frac{1}{3} + 1 - 3\right)$$

$$E = \frac{32}{3}\ \tau.\mu.$$