Κλιμακωμένη προσέγγιση ολοκληρωμάτων

1. $\int dx$
2. $\int (x^3 + x^{-4})dx$
3. $\int \sqrt[3]{x^4}\,dx$
4. $\int \frac{x^2}{\sqrt{x}}dx$
5. $\int x^3(x^2 + 2x - 3)dx$
6. $\int \frac{4}{x}dx$
7. $\int (e^{3x} + e^{-2x})dx$
8. $\int e^4 dx$
9. $\int (\sigma\upsilon\nu 7x + \eta\mu 3x)dx$
10. $\int \tau\varepsilon\mu^2 \frac{3x}{2}dx$
11. $\int \frac{dx}{\sqrt{x^2 - 1}}$
12. $\int \frac{dx}{1 - x^2}$
13. $\int \frac{dx}{\sqrt{1 - x^2}}$
14. $\int 2x(x^2 + 5)^4 dx$
15. $\int e^x(e^x + 3)^{-2} dx$
16. $\int \frac{\ln^5 x}{x}dx$
17. $\int \eta\mu^6 x.\sigma\upsilon\nu x\, dx$
18. $\int \tau\varepsilon\mu x.\varepsilon\varphi x\, dx$
19. $\int \frac{dx}{2x + 3}$
20. $\int \frac{3dx}{4 - 5x}$
21. $\int \frac{\sigma\upsilon\nu x}{1 + \eta\mu x}dx$
22. $\int \frac{\eta\mu 2x}{1 + \sigma\upsilon\nu^2 x}dx$
23. $\int \frac{dx}{\sqrt{2x + 7}}$
24. $\int \frac{e^x}{\sqrt{e^x + 5}}dx$
25. $\int \frac{dx}{x\sqrt{2 + \ln x}}$
26. $\int x\sqrt{x + 8}\, dx$
27. $\int x^3\sqrt{x^2 + 1}\, dx$
28. $\int x(x - 2)^3 dx$
29. $\int \frac{\varepsilon\varphi x}{\tau\varepsilon\mu^5 x}dx$
30. $\int \frac{x}{x + 3}dx$
31. $\int \frac{x^4 + 4}{x^2 + 1}dx$
32. $\int (3x + 4)^5 dx$
33. $\int \sqrt{8x + 7}\, dx$
34. $\int \frac{dx}{\sqrt[5]{(2x + 3)^3}}$
35. $\int \frac{2x + 7}{x^2 + 1}dx$
36. $\int \frac{3x + 4}{\sqrt{x^2 + 1}}dx$
37. $\int \frac{\sigma\upsilon\nu x}{1 + \eta\mu^2 x}dx$
38. $\int \frac{dx}{e^x + 3}$
39. $\int \frac{dx}{4 + 3e^{-x}}$
40. $\int x.e^{-x} dx$
41. $\int \ln x\, dx$
42. $\int x^2.\ln x\, dx$
43. $\int x.\tau o\xi\varepsilon\varphi x\, dx$
44. $\int \ln^2 x\cdot dx$
45. $\int e^x.\eta\mu x\, dx$
46. $\int \frac{\tau o\xi\eta\mu x}{\sqrt{x + 1}}dx$
47. $\int x^5.e^{x^3} dx$
48. $\int e^{\sqrt{x}} dx$
49. $\int \eta\mu^2 7x\, dx$
50. $\int \sigma\upsilon\nu^2 3x\, dx$
51. $\int \varepsilon\varphi^5 5x\, dx$
52. $\int \sigma\tau\varepsilon\mu^2 \frac{5x}{2}dx$
53. $\int \eta\mu 7x\, \sigma\upsilon\nu 3x\, dx$

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54. $\int \frac{(\text{τοξεφ}x)^5}{1+x^2}dx$

55. $\int \sqrt{1-\text{συν}x}\,dx$

56. $\int \sqrt{1+\text{συν}x}\,dx$

57. $\int \sqrt{1+\text{ημ}2x}\,dx$

58. $\int \text{συν}^3x\,dx$

59. $\int \text{εφ}^3x\,dx$

60. $\int \text{ημ}^4x\,dx$

61. $\int \text{συν}^4x\,dx$

62. $\int \text{εφ}^4x\,dx$

63. $\int \frac{dx}{x^2+9}$

64. $\int \frac{dx}{(x-2)^2+16}$

65. $\int \frac{dx}{(2x+3)^2+25}$

66. $\int \frac{dx}{\sqrt{x^2-25}}$

67. $\int \frac{dx}{\sqrt{4x^2+9}}$

68. $\int \frac{dx}{x^2+6x+13}$

69. $\int \frac{dx}{\sqrt{x^2+8x+20}}$

70. $\int \frac{dx}{\sqrt{x(4-x)}}$

71. $\int \frac{x^2}{1+x^6}dx$

72. $\int \frac{dx}{16-x^2}$

73. $\int \frac{dx}{9x^2-25}$

74. $\int \frac{1}{x\sqrt{1-\ln^2 x}}dx$

75. $\int \frac{\text{ημ}x}{\sqrt{\text{συν}^2x+1}}dx$

76. $\int \frac{e^x}{\sqrt{e^{2x}-1}}dx$

77. $\int \frac{2^x}{\sqrt{4^x+1}}dx$

78. $\int \frac{2x+6}{x^2+4x+8}dx$

79. $\int \frac{3x+11}{x^2+8x+17}dx$

80. $\int \frac{5x+3}{\sqrt{x^2+6x+13}}dx$

81. $\int \frac{3x-5}{\sqrt{x(4-x)}}dx$

82. $\int \frac{7x+11}{x^2+8x+12}dx$

83. $\int \frac{3x}{\sqrt{12x-9x^2-2}}dx$

84. $\int \frac{3+x}{(1-x)(1+3x)}dx$

85. $\int \frac{x^2+2x-1}{(x+1)(x^2+1)}dx$

86. $\int \frac{x^2+x-3}{(x-2)^2(x-1)}dx$

87. $\int \sqrt{9-x^2}\,dx$

88. $\int \sqrt{16+x^2}\,dx$

89. $\int \sqrt{6x-x^2-8}\,dx$

90. $\int \sqrt{9x^2-4}\,dx$

91. $\int \frac{x^2}{\sqrt{x^2+4}}dx$

92. $\int \frac{dx}{\text{συν}2x-3\text{ημ}^2x}$

93. $\int \frac{dx}{1+\text{ημ}x+\text{συν}x}$

94. $\int \frac{3dx}{5+4\text{συν}x+3\text{ημ}x}$

95. $\int \text{τεμ}x\,dx$

95. $\int \text{στεμ}x\,dx$

97. $\int \frac{dx}{x\sqrt{x^2+1}}$

98. $\int \frac{dx}{x^2\sqrt{1+x^2}}$

99. $\int \sqrt{\frac{x-2}{x+2}}\,dx$

100. $\int \frac{\text{ημ}^3x}{\text{συν}^2x}dx$

101. $\int \frac{\text{ημ}^4x}{\text{συν}^6x}dx$

102. $\int \frac{dx}{1+\sqrt{x-2}}$

103. $\int \frac{e^{-\frac{2}{x}}}{x^3}dx$

104. $\int (\text{εφ}^5x+\text{εφ}^7x)dx$

105. $\int \frac{dx}{1+\text{σφ}x}$

106. $\int \text{τεμ}^3x\,dx$

107. $\int e^{3-2\ln x}dx$

108. $\int \ln(x+\sqrt{x^2+1})dx$

109. $\int \sqrt{e^x+1}\,dx$

110. $\int \frac{dx}{\sqrt{x-x^2}}$

111. $\int \frac{x^3}{x^2-4}dx$

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ΛΥΣΕΙΣ ΑΣΚΗΣΕΩΝ ΣΤΑ ΟΛΟΚΛΗΡΩΜΑΤΑ

1) $\int dx = x + c$

2) $\int (x^3 + x^{-4})\,dx = \frac{x^4}{4} + \frac{x^{-3}}{3} + c$

3) $\int \sqrt[3]{x^4}\,dx = \int x^{\frac{4}{3}}\,dx = \frac{x^{\frac{7}{3}}}{\frac{7}{3}} + c = \frac{3}{7}\sqrt[3]{x^7} + c$

4) $\int \frac{x^2}{\sqrt{x}}\,dx = \int x^2 \cdot x^{-\frac{1}{2}}\,dx = \int x^{\frac{3}{2}}\,dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c = \frac{2}{5}\sqrt{x^5} + c$

5) $\int x^3(x^2+2x-3)\,dx = \int (x^5 + 2x^4 - 3x^3)\,dx = \frac{x^6}{6} + \frac{2x^5}{5} - \frac{3x^4}{4} + c$

6) $\int \frac{4}{x}\,dx = 4\ln|x| + c$

7) $\int (e^{3x} + e^{-2x})\,dx = \frac{e^{3x}}{3} - \frac{e^{-2x}}{2} + c$

8) $\int e^4\,dx = e^4 x + c$

9) $\int (\text{συν}7x + \text{ημ}3x)\,dx = \frac{\text{ημ}7x}{7} - \frac{\text{συν}3x}{3} + c$

10) $\int \text{τεμ}^2 \frac{3x}{2}\,dx = \frac{2}{3}\,\text{εφ}\frac{3x}{2} + c$

$\boxed{\text{εφ}^2 t = \text{τεμ}^2 t - 1}$

11) $\int \frac{dx}{\sqrt{x^2-1}} = \int \frac{\text{τεμ}t \cdot \cancel{\text{εφ}t}\,dt}{\cancel{\text{εφ}t}} =$

$x = \text{τεμ}t$

$dx = \text{τεμ}t \cdot \text{εφ}t\,dt$

$\text{συν}t = \frac{1}{x}$ $\quad$ $\text{ημ}t = \sqrt{1 - \frac{1}{x^2}} = \frac{1}{x}\sqrt{x^2-1}$

$= \int \text{τεμ}t\,dt = \ln|\text{τεμ}t + \text{εφ}t| + c =$

$= \ln\left|x + \frac{\text{ημ}t}{\text{συν}t}\right| + c = \ln\left|x + \frac{\cancel{\frac{1}{x}}\sqrt{x^2-1}}{\cancel{\frac{1}{x}}}\right| + c = \ln\left|x + \sqrt{x^2-1}\right| + c$

12) $\int \frac{dx}{1-x^2} = \frac{1}{2}\int \frac{dx}{1-x} + \frac{1}{2}\int \frac{dx}{1+x} = \frac{1}{2}\ln|1-x| + \frac{1}{2}\ln|1+x| + c = \ln\left|\sqrt{\frac{1-x}{1+x}}\right| + c$

$\frac{1}{1-x^2} = \frac{A}{1-x} + \frac{B}{1+x} \Leftrightarrow 1 = A(1+x) + B(1-x)$

για $x = 1 \Rightarrow A = \frac{1}{2}$ $\qquad$ για $x = -1 \Leftrightarrow B = \frac{1}{2}$

13) $\int \frac{dx}{\sqrt{1-x^2}} = \text{τοξημ}\, x + c$

14) $\int 2x(x^2+5)^4 dx = \int (x^2+5)^4 d(x^2+5) =$ $\boxed{d(x^2+5) = 2x\,dx}$

$= \frac{(x^2+5)^5}{5} + c$

15) $\int e^x(e^x+3)^{-2} dx = \int (e^x+3)^{-2} d(e^x+3) =$ $\boxed{d(e^x+3) = e^x dx}$

$= \frac{(e^x+3)^{-1}}{-1} + c = -\frac{1}{e^x+3} + c$

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16) $\int \frac{\ln^5 x}{x} dx = \int \ln^5 x \, d(\ln x) = \frac{\ln^6 x}{6} + c$

17) $\int \text{ημ}^6 x\, \text{συν} x\, dx = \int \text{ημ}^6 x\, d(\text{ημ} x) = \frac{\text{ημ}^7 x}{7} + c$

18) $\int \text{τεμ} x \cdot \text{εφ} x\, dx = \text{τεμ} x + c$

19) $\int \frac{dx}{2x+3} =$ $\boxed{d(2x+3) = 2dx}$

$= \frac{1}{2}\int \frac{d(2x+3)}{2x+3} = \frac{1}{2} \ln|2x+3| + c$

20) $\int \frac{3dx}{4-5x} = -\frac{3}{5}\int \frac{d(4-5x)}{4-5x} =$ $\boxed{d(4-5x) = -5dx}$

$= -\frac{3}{5} \ln|4-5x| + c$

21) $\int \frac{\text{συν} x}{1+\text{ημ} x} dx = \int \frac{d(1+\text{ημ} x)}{1+\text{ημ} x} = \ln|1+\text{ημ} x| + c$ $\boxed{d(1+\text{ημ} x) = \text{συν} x\, dx}$

22) $\int \frac{\text{ημ} 2x}{1+\text{συν}^2 x} dx = -\int \frac{d(1+\text{συν}^2 x)}{1+\text{συν}^2 x} =$ $d(1+\text{συν}^2 x) = -2\,\text{συν} x\, \text{ημ} x\, dx$
$= -\text{ημ} 2x\, dx$

$= -\ln|1+\text{συν}^2 x| + c$

23) $\int \frac{dx}{\sqrt{2x+7}} = \int d(\sqrt{2x+7}) =$ $d(\sqrt{2x+7}) = \frac{\cancel{2}}{\cancel{2}\sqrt{2x+7}} dx$

$= \sqrt{2x+7} + c$

24) $\displaystyle\int \frac{e^x}{\sqrt{e^x+5}}\,dx = 2\int d(\sqrt{e^x+5}) =$ $\qquad d(\sqrt{e^x+5}) = \dfrac{e^x\,dx}{2\sqrt{e^x+5}}$

$= 2\sqrt{e^x+5} + c$

25) $\displaystyle\int \frac{dx}{x\cdot\sqrt{2+\ln x}} = 2\int d(\sqrt{2+\ln x}) =$ $\qquad d(\sqrt{2+\ln x}) = \dfrac{dx}{2x\sqrt{2+\ln x}}$

$= 2\sqrt{2+\ln x} + c$

26) $\displaystyle\int (x\cdot\sqrt{x+8})\,dx =$ $\qquad \sqrt{x+8} = u \Rightarrow x+8 = u^2$

$dx = 2u\,du$

$\displaystyle= \int (u^2-8)\cdot u\cdot 2u\,du = 2\int u^4\,du - 16\int u^2\,du =$

$\displaystyle= 2\frac{u^5}{5} - 16\frac{u^3}{3} + c$

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27) $\displaystyle\int (x^3\sqrt{x^2+1})\,dx =$ $\qquad \sqrt{x^2+1} = u \Leftrightarrow x^2+1 = u^2$

$\cancel{2}x\,dx = \cancel{2}u\,du$

$\displaystyle= \int x^2\cdot\sqrt{x^2+1}\cdot x\,dx = \int (u^2-1)\cdot u\cdot u\,du =$

$\displaystyle= \int u^4\,du - \int u^2\,du = \frac{u^5}{5} - \frac{u^3}{3} + c$

28) $\displaystyle\int x(x-2)^3\,dx = \int (u+2)\cdot u\,du =$ $\qquad x-2 = u$

$dx = du$

$\displaystyle= \int u^2\,du + 2\int u\,du = \frac{u^3}{3} + \cancel{2}\frac{u^2}{\cancel{2}} + c = \frac{u^3}{3} + u^2 + c$

29) $\displaystyle\int \frac{\text{εφ}x}{\text{τεμ}^5 x}\,dx = \int \frac{du}{u^6} =$ $\qquad \text{τεμ}x = u$

$\text{τεμ}x\cdot\text{εφ}x\,dx = du$

$\displaystyle= \int u^{-6}\,du = \frac{u^{-5}}{-5} + c = -\frac{1}{5u^5} + c = -\frac{1}{5\,\text{τεμ}^5 x} + c$

30) $\displaystyle\int \frac{x}{x+3}\,dx = \int \frac{x+3-3}{x+3}\,dx = \int dx - 3\int \frac{dx}{x+3} =$

$= x - 3\ln|x+3| + c$

31) $\int \frac{x^4+4}{x^2+1}\,dx =$

$$\begin{array}{r|l} x^4+4 & x^2+1 \\ \hline -x^4-x^2 & x^2-1 \\ -x^2+4 & \\ +x^2+1 & \\ \hline 5 & \end{array}$$

$= \int \left(x^2-1+\frac{5}{x^2+1}\right)dx =$

$= \int (x^2-1)\,dx + 5\int \frac{dx}{x^2+1} = \frac{x^3}{3} - x + 5\,\text{τοξεφ}\,x + c$

32) $\int (3x+4)^5\,dx = \frac{1}{3}\int (3x+4)^5\,d(3x+4) = \frac{(3x+4)^6}{18} + c$

33) $\int \left(\sqrt{8x+7}\right)dx =$

$\sqrt{8x+7} = u \Leftrightarrow$

$8x+7 = u^2 \Leftrightarrow 8\,dx = 2u\,du$

$4\,dx = u\,du$

$= \frac{1}{4}\int u\cdot u\,du = \frac{u^3}{12} + c$

34) $\int \frac{dx}{\sqrt[5]{(2x+3)^3}} = \int (2x+3)^{-\frac{3}{5}}\,dx = \frac{1}{2}\int (2x+3)^{-\frac{3}{5}}\,d(2x+3) =$

$= \frac{1}{2}\,\frac{(2x+3)^{\frac{2}{5}}}{\frac{2}{5}} = \frac{5}{4}(2x+3)^{\frac{2}{5}} + c = \frac{5}{4}\sqrt[5]{(2x+3)^2} + c$

35) $\int \frac{2x-7}{x^2+1}\,dx = 2\int \frac{x\,dx}{x^2+1} - 7\int \frac{dx}{x^2+1} =$

$= \int \frac{d(x^2+1)}{x^2+1} - 7\int \frac{dx}{x^2+1} = \ln(x^2+1) - 7\,\text{τοξεφ}\,x + c$

36) $\int \frac{3x+4}{\sqrt{x^2+1}}\,dx = \int \frac{3x}{\sqrt{x^2+1}}\,dx + 4\int \frac{dx}{\sqrt{x^2+1}} =$

$= 3\int d\left(\sqrt{x^2+1}\right) + 4\int \frac{\text{τεμ}^{\cancel{2}} t\,dt}{\cancel{\text{τεμ} t}} =$

$x = \text{εφ}\,t$

$dx = \text{τεμ}^2 t\,dt$

$\text{εφ}^2 t + 1 = \text{τεμ}^2 t$

$= 3\sqrt{x^2+1} + 4\ln|\text{τεμ}\,t + \text{εφ}\,t| + c$

$= 3\sqrt{x^2+1} + 4\ln\left|\sqrt{x^2+1} + x\right| + c$

37) $\int \frac{\sigma v \nu x}{1+\eta\mu^2 x}dx = \int \frac{du}{1+u^2} =$ $\quad$ Θέτω $\eta\mu x = u \Leftrightarrow$ $\sigma v\nu x\,dx = du$

$= \tau o \xi \varepsilon \varphi u + c = \tau o \xi \varepsilon \varphi\, \eta\mu x + c$

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38) $\int \frac{dx}{e^x+3} = \int \frac{du}{(u-3)\cdot u} =$ $\quad$ Θέτω $e^x+3 = u$, $e^x dx = du$

$= \frac{1}{3}\int \frac{du}{u-3} - \frac{1}{3}\int \frac{du}{u} =$ $\quad$ $\frac{1}{(u-3)\cdot u} = \frac{A}{u-3} + \frac{B}{u} \Leftrightarrow$

$= \frac{1}{3}\ln|u-3| - \frac{1}{3}\ln|u| + c$ $\quad$ $1 = Au + B(u-3)$

για $u=0 \Leftrightarrow B = -\frac{1}{3}$

$= \frac{1}{3}\ln|e^x| - \frac{1}{3}\ln(e^x+3) + c$ $\quad$ για $u=3 \Leftrightarrow A = \frac{1}{3}$

$= \frac{1}{3}\ln\left|\frac{e^x}{e^x+3}\right| + c$

β' τρόπος $\int \frac{dx}{e^x+3} = \int \frac{dx}{3+\frac{1}{e^{-x}}} = \int \frac{dx}{\frac{3e^{-x}+1}{e^{-x}}} = \int \frac{e^{-x}dx}{3e^{-x}+1} =$

$= -\frac{1}{3}\int \frac{d(3e^{-x}+1)}{3e^{-x}+1} =$ $\quad$ $\boxed{d(3e^{-x}+1) = -3e^{-x}dx}$

$= -\frac{1}{3}\ln|3e^{-x}+1| + c = \frac{1}{3}\ln\left|\frac{e^x}{e^x+3}\right| + c$

39) $\int \frac{dx}{4+3e^{-x}} = \int \frac{dx}{4+\frac{3}{e^x}} = \int \frac{e^x dx}{4e^x+3} = \frac{1}{4}\int \frac{d(4e^x+3)}{4e^x+3} =$

$= \frac{1}{4}\ln|4e^x+3| + c$

40) $\int x e^{-x} dx = -\int x\, d(e^{-x}) = -x\cdot e^{-x} + \int e^{-x}dx =$

$= -xe^{-x} - e^{-x} + c$

41) $\int \ln x\, dx = x\cdot\ln x - \int x\, d(\ln x) = x\ln x - \int dx =$

$= x\ln x - x + c$

42) $\int x^2\cdot \ln x\,dx = \frac{1}{3}\int \ln x\, d(x^3) = \frac{1}{3}x^3\ln x - \int x^3 d(\ln x) =$

$= \frac{1}{3}x^3\ln x - \int x^2\,dx = \frac{1}{3}x^3\ln x - \frac{x^3}{3} + c$

43) $\int x\,\text{τοξεφ}x\,dx = \frac{1}{2}\int \text{τοξεφ}x\, d(x^2) =$

$= \frac{1}{2}x^2\cdot\text{τοξεφ}x - \frac{1}{2}\int \frac{x^2}{1+x^2}dx = \frac{1}{2}x^2\,\text{τοξεφ}x - \frac{1}{2}\int\frac{x^2+1-1}{1+x^2}dx$

$= \frac{1}{2}x^2\,\text{τοξεφ}x - \frac{1}{2}\int\frac{\cancel{x^2+1}}{\cancel{1+x^2}}dx + \frac{1}{2}\int\frac{dx}{1+x^2} =$

$= \frac{1}{2}x^2\,\text{τοξεφ}x - \frac{1}{2}x + \frac{1}{2}\,\text{τοξεφ}x + c$

44) $\int \ln^2 x\,dx = x\ln^2 x - \int x\cdot d(\ln^2 x) =$

$= x\cdot\ln^2 x - 2\int \cancel{x}\cdot\frac{\ln x}{\cancel{x}}dx = x\cdot\ln^2 x - 2x\ln x + 2\int x\,d(\ln x) =$

$= x\cdot\ln^2 x - 2x\cdot\ln x + 2\int dx = x\ln^2 x - 2x\ln x + 2x + c$

45) $I = \int e^x\,\text{ημ}x\,dx = \int \text{ημ}x\,d(e^x) = e^x\cdot\text{ημ}x - \int e^x d(\text{ημ}x) =$

$= e^x\,\text{ημ}x - \int e^x\cdot\text{συν}x\,dx = e^x\cdot\text{ημ}x - \int\text{συν}x\,d(e^x) =$

$= e^x\cdot\text{ημ}x - e^x\,\text{συν}x + \int e^x d(\text{συν}x) = e^x\cdot\text{ημ}x - e^x\text{συν}x - \int e^x\text{ημ}x\,dx$

$\Leftrightarrow I = e^x\cdot\text{ημ}x - e^x\text{συν}x - I \Leftrightarrow I = \frac{e^x}{2}(\text{ημ}x - \text{συν}x) + c$

46) $\int \frac{\text{τοξημ}x}{\sqrt{x^2+1}}dx = \int \text{τοξημ}x\,d(\text{τοξημ}x) = \frac{\text{τοξημ}^2x}{2} + c$

47) $\int x^5 e^{x^3}dx = \int x^3\cdot e^{x^3}\cdot x^2dx =$ Θέτω $x^3 = u$

$3x^2dx = du$

$= \frac{1}{3}\int u\cdot e^u\,du = \frac{1}{3}\int u\,d(e^u) = \frac{1}{3}u\cdot e^u - \frac{1}{3}\int e^u du =$

48) $\int e^{\sqrt{x}}dx = 2\int e^{u} u\,du =$ $\qquad \sqrt{x}=u \Leftrightarrow x=u^2$, $dx = 2u\,du$

$= 2\int u\,d(e^{u}) = 2ue^{u} - 2\int e^{u}du =$

$= 2\cdot u\cdot e^{u} - 2e^{u} + c = 2\sqrt{x}\cdot e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$

49) $\int \eta\mu^2 7x\,dx = \int\left(\frac{1-\sigma\upsilon\nu 14x}{2}\right)dx = \frac{x}{2} - \frac{\eta\mu 14x}{28} + c$

50) $\int \sigma\upsilon\nu^2 3x\,dx = \int\left(\frac{1+\sigma\upsilon\nu 6x}{2}\right)dx = \frac{x}{2} + \frac{\eta\mu 6x}{12} + c$

51) $I_5 = \int \varepsilon\varphi^5 x\,dx$

Γενική μορφή $I_\nu = \int \varepsilon\varphi^\nu x\,dx = \int \varepsilon\varphi^2 x\cdot\varepsilon\varphi^{\nu-2}x\,dx =$

$= \int(1+\varepsilon\varphi^2 x - 1)\,\varepsilon\varphi^{\nu-2}x\,dx = \int (1+\varepsilon\varphi^2 x)\,\varepsilon\varphi^{\nu-2}x\,dx - \int \varepsilon\varphi^{\nu-2}x\,dx =$

$= \int \varepsilon\varphi^{\nu-2}x\,d(\varepsilon\varphi x) - \int \varepsilon\varphi^{\nu-2}x\,dx = \frac{\varepsilon\varphi^{\nu-1}x}{\nu-1} - I_{\nu-2}$

$\boxed{\nu=5}$

$I_5 = \int \varepsilon\varphi^5 x\,dx = \frac{\varepsilon\varphi^4 x}{4} - \int \varepsilon\varphi^3 x\,dx =$

$= \frac{\varepsilon\varphi^4 x}{4} - \frac{\varepsilon\varphi^2 x}{4} + \int \varepsilon\varphi x\,dx = \frac{\varepsilon\varphi^4 x}{4} - \frac{\varepsilon\varphi^2 x}{2} - \ln|\sigma\upsilon\nu x| + c$

52) $\int \sigma\tau\varepsilon\varphi^2 \frac{5x}{2}\,dx = \frac{2}{5}\varepsilon\varphi\frac{5x}{2} + c$

53) $\int \eta\mu 7x\cdot\sigma\upsilon\nu 3x\,dx = \frac{1}{2}\int \eta\mu 10x\,dx + \frac{1}{2}\int \eta\mu 4x\,dx =$

$= -\frac{1}{2}\frac{\sigma\upsilon\nu 10x}{10} - \frac{1}{2}\frac{\sigma\upsilon\nu 4x}{4} + c$

54) $\int \frac{\tau o\xi\varepsilon\varphi^5 x}{1+x^2}\,dx = \int \tau o\xi\varepsilon\varphi^5 x\,d(\tau o\xi\varepsilon\varphi x) = \frac{\tau o\xi\varepsilon\varphi^6 x}{6} + c$

55) $\int\sqrt{1-\sigma\upsilon\nu x}\,dx=\int\sqrt{2\eta\mu^2\frac{x}{2}}\,dx=\sqrt{2}\int\eta\mu\frac{x}{2}\,dx=$

$=-2\sqrt{2}\,\sigma\upsilon\nu\frac{x}{2}+c$

56) $\int\sqrt{1+\sigma\upsilon\nu x}\,dx=\int\sqrt{2\sigma\upsilon\nu^2\frac{x}{2}}\,dx=\sqrt{2}\int\sigma\upsilon\nu\frac{x}{2}\,dx=$

$=2\sqrt{2}\cdot\eta\mu\frac{x}{2}+c$

57) $\int\sqrt{1+\eta\mu 2x}\,dx=\int\sqrt{\eta\mu^2x+\sigma\upsilon\nu^2x+2\eta\mu x\,\sigma\upsilon\nu x}\,dx=$

$=\int\sqrt{(\eta\mu x+\sigma\upsilon\nu x)^2}\,dx=\pm\int(\eta\mu x+\sigma\upsilon\nu x)\,dx=\pm(\sigma\upsilon\nu x+\eta\mu x)+c$

58) $\int\sigma\upsilon\nu^3x\,dx=\int\sigma\upsilon\nu^2x\cdot\sigma\upsilon\nu x\,dx=\int(1-\eta\mu^2x)\,d(\eta\mu x)=$

$=\eta\mu x-\frac{\eta\mu^3x}{3}+c$

59) $\int\varepsilon\varphi^3x\,dx=\int\varepsilon\varphi x\cdot\varepsilon\varphi^2x\,dx=\int(1+\varepsilon\varphi^2x-1)\,\varepsilon\varphi x\,dx=$

$=\int\varepsilon\varphi x(1+\varepsilon\varphi^2x)\,dx-\int\varepsilon\varphi x\,dx=\int\varepsilon\varphi x\,d(\varepsilon\varphi x)-\int\varepsilon\varphi x\,dx=$

$=\frac{\varepsilon\varphi^2x}{2}+\ln|\sigma\upsilon\nu x|+c$

60) $\int\eta\mu^4x\,dx=\int(\eta\mu^2x)^2\,dx=\int\left(\frac{1-\sigma\upsilon\nu 2x}{2}\right)^2dx=$

$=\frac{1}{4}\int(1-2\sigma\upsilon\nu 2x+\sigma\upsilon\nu^2 2x)\,dx=\frac{1}{4}\int\left[1-\sigma\upsilon\nu 2x+\left(\frac{1+\sigma\upsilon\nu 4x}{2}\right)\right]dx$

$=\frac{1}{4}\left[x-\cancel{\frac{2\eta\mu 2x}{2}}+\frac{x}{2}+\frac{\eta\mu 4x}{8}\right.+c$

61) $\int\sigma\upsilon\nu^4x\,dx=\int(\sigma\upsilon\nu^2x)^2\,dx=\int\left(\frac{1+\sigma\upsilon\nu 2x}{2}\right)^2dx=$

$=\frac{1}{4}\int(1+2\sigma\upsilon\nu 2x+\sigma\upsilon\nu^2 2x)\,dx=\frac{1}{4}\int\left[1+\sigma\upsilon\nu 2x+\left(\frac{1+\sigma\upsilon\nu 2x}{2}\right)\right]dx$

$=\frac{1}{4}\left[x+\eta\mu 2x+\frac{x}{2}+\frac{\eta\mu 4x}{8}\right.+c$

62) $\int εφ^4x\,dx = \int εφ^2x\,(εφ^2x)\,dx = \int (τεμ^2x-1)\,εφ^2x\,dx =$

$= \int τεμ^2x\,εφ^2x\,dx - \int εφ^2x\,dx = \frac{εφ^3x}{3} - \int (τεμ^2x-1)\,dx =$

$= \frac{εφ^3x}{3} - εφx + x + c$

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63) $\int \frac{dx}{x^2+9} = \int \frac{dx}{x^2+3^2} = \frac{1}{3}\,τοξεφ\,\frac{x}{3} + c$

64) $\int \frac{dx}{(x-2)^2+16} = \int \frac{dx}{(x-2)^2+4^2} = \frac{1}{4}\,τοξεφ\,\frac{x-2}{4} + c$

65) $\int \frac{dx}{(2x+3)^2+25} = \int \frac{dx}{(2x+3)^2+5^2} = \frac{1}{5}\,τοξεφ\,\frac{2x+3}{5} + c$

66) $\int \frac{dx}{\sqrt{x^2-25}} = \int \frac{5\,τεμt \cdot εφt\,dt}{\sqrt{25τεμ^2t-25}}$

$\boxed{\begin{array}{l} \text{Θέτω } x = 5\,τεμt \\ dx = 5\,τεμt \cdot εφt \end{array}}$

$= \frac{5}{5}\int \frac{τεμt \cdot εφt\,dt}{\sqrt{εφ^2t}} = \int τεμt\,dt = \ln|τεμt + εφt| =$

$= \ln\left|\frac{x}{5} + \sqrt{\frac{x}{25}-1}\right| + c$

$\boxed{\begin{array}{l} εφ^2t+1 = τεμ^2t \\ εφt = \sqrt{τεμ^2t-1} \end{array}}$

67) $\int \frac{dx}{\sqrt{4x^2+9}} = \frac{3}{2}\int \frac{τεμ^2t\,dt}{\sqrt{4\cdot\frac{9}{4}εφ^2t+9}} =$ Θέτω $x = \frac{3}{2}εφt$, $dx = \frac{3}{2}τεμ^2t\,dt$

$= \frac{3}{2}\int \frac{τεμ^2t\,dt}{\sqrt{9(εφ^2t+1)}} = \frac{1}{2}\int \frac{τεμ^2t\,dt}{τεμt} = \frac{1}{2}\int τεμt\,dt =$

$= \frac{1}{2}\ln|τεμt + εφt| + c = \frac{1}{2}\ln\left|\sqrt{\frac{4}{9}x^2+1} + \frac{2}{3}x\right| + c$

68) $\displaystyle\int \frac{dx}{x^2+6x+13} = \int \frac{dx}{(x+3)^2+4} = \frac{1}{2}\,\text{τοξεφ}\,\frac{x+3}{2} + c$

69) $\displaystyle\int \frac{dx}{\sqrt{x^2+8x+20}} = \int \frac{dx}{\sqrt{(x+4)^2+4}} =$

$\boxed{\text{Θέτω } x+4 = 2\,\text{εφ}t \quad dx = 2\,\text{τεμ}^2t\,dt}$

$\displaystyle = 2\int \frac{\text{τεμ}^2t\,dt}{\sqrt{4\text{εφ}^2t+4}} = \int \frac{\text{τεμ}^{\cancel{2}}t\,dt}{\cancel{\text{τεμ}t}} =$

$\displaystyle = \int \text{τεμ}t\,dt = \ln|\text{τεμ}t + \text{εφ}t| + c =$

$\displaystyle = \ln\left|\sqrt{\left(\frac{x+4}{2}\right)^2+1} + \frac{x+4}{2}\right| + c$

70) $\displaystyle\int \frac{dx}{\sqrt{x(4-x)}} = \int \frac{dx}{\sqrt{4x-x^2}} = \int \frac{dx}{\sqrt{-(x^2-4x+4-4)}} =$

$\displaystyle = \int \frac{dx}{\sqrt{-(x-2)^2+4}} = \int \frac{dx}{\sqrt{4-(x-2)^2}} = \text{τοξημ}\left(\frac{x-2}{2}\right) + c$

71) $\displaystyle\int \frac{x^2}{1+x^6}\,dx = \frac{1}{3}\int \frac{du}{1+u^2} =$ $\quad x^3 = u \Leftrightarrow 3x^2dx = du$

$\displaystyle = \frac{1}{3}\,\text{τοξεφ}\,u + c = \frac{1}{3}\,\text{τοξεφ}\,x^3 + c$

72) $\displaystyle\int \frac{dx}{16-x^2} = \int \frac{dx}{4^2-x^2} = -\int \frac{dx}{x^2-4^2} = -\frac{1}{8}\ln\left|\frac{x-4}{x+4}\right| + c$

73) $\displaystyle\int \frac{dx}{9x^2-25} = \frac{1}{9}\int \frac{dx}{x^2-\left(\frac{5}{3}\right)^2} =$

$\boxed{\text{Τύπος} \quad \displaystyle\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c}$

$\displaystyle = \frac{1}{9\cdot 2\,\frac{5}{3}}\ln\left|\frac{x-\frac{5}{3}}{x+\frac{5}{3}}\right| + c = \frac{1}{30}\ln\left|\frac{3x-5}{3x+5}\right| + c$

74) $\int \frac{dx}{x\sqrt{1-\ln^2 x}} =$ Θέτω $\ln x = u$, $\frac{dx}{x} = du$

$= \int \frac{du}{\sqrt{1-u^2}} = τοξημ\, u + c = τοξημ(\ln|x|) + c$

75) $\int \frac{ημx}{\sqrt{συν^2x+1}}\,dx =$ Θέτω $συνx = u$, $-ημx\,dx = du$

$= -\int \frac{du}{\sqrt{u^2+1}} = -\int \frac{τεμ^2t\,dt}{\sqrt{εφ^2t+1}} =$ Θέτω $u = εφt$, $du = τεμ^2t\,dt$

$= -\int τεμ\,t\,dt = -\ln|τεμt + εφt| + c =$

$= -\ln|\sqrt{u^2+1} + u| + c = -\ln|\sqrt{συν^2x+1} + συνx| + c$

76) $\int \frac{e^x}{\sqrt{e^{2x}-1}}\,dx =$ $e^x = u$ άρα $e^x dx = du$

$= \int \frac{du}{\sqrt{u^2-1}} = \int \frac{τεμt \cdot εφt\,dt}{\sqrt{τεμ^2t-1}} =$ $u = τεμt$, $du = τεμt \cdot εφt\,dt$

$= \int τεμt\,dt = \ln|τεμt + εφt| + c = \ln|u + \sqrt{u^2-1}| + c$

$= \ln|e^x + \sqrt{e^{2x}-1}| + c$

77) $\int \frac{2^x}{\sqrt{4^x+1}}\,dx = \frac{1}{\ln 2}\int \frac{du}{\sqrt{u^2+1}} =$ $2^x = u$ άρα $2^x \ln 2\,dx = du$

$= -\ln|\sqrt{u^2+1} + u| + c = -\ln|\sqrt{4^x+1} + 2^x| + c$

78) $\int \frac{2x+6}{x^2+4x+8}\,dx = \int \frac{2x+4+2}{x^2+4x+8}\,dx =$ $\boxed{d(x^2+4x+8) = (2x+4)dx}$

$= \int \frac{2x+4}{\ldots}dx + 2\int \frac{dx}{\ldots} = \ln|x^2+4x+8| + 2\,τοξεφ\,(x+2) + c$

79) $\displaystyle\int \frac{3x+11}{x^2+8x+17}\,dx = \int \frac{\frac{3}{2}(2x+8)-1}{x^2+8x+17}\,dx =$ $\quad d(x^2+8x+17)=(2x+8)dx$

$\displaystyle = \frac{3}{2}\int \frac{2x+8}{x^2+8x+17}\,dx - \int \frac{dx}{x^2+8x+17} =$

$\displaystyle = \frac{3}{2}\int \frac{d(x^2+8x+17)}{x^2+8x+17} - \int \frac{dx}{(x+4)^2+1} =$

$\displaystyle = \frac{3}{2}\ln|x^2+8x+17| - \text{τοξεφ}(x+4) + C$

80) $\displaystyle\int \frac{5x+3}{\sqrt{x^2+6x+13}}\,dx =$ $\quad d(x^2+6x+13)=(2x+6)dx$

$\displaystyle = \int \frac{\frac{5}{2}(2x+6)\cancel{+}12}{\sqrt{x^2+6x+13}}\,dx = \frac{5}{2}\int \frac{d(x^2+6x+13)}{\sqrt{x^2+6x+13}} - 12\int \frac{dx}{\sqrt{(x+3)^2+2^2}}$

$\displaystyle = 5\sqrt{x^2+6x+13} - 12\int \frac{2\,\text{τεμ}^2 t\,dt}{\sqrt{4(\text{εφ}^2 t+1)}} =$ $\quad x+3 = 2\,\text{εφ}\,t$, $\quad dx = 2\,\text{τεμ}^2 t\,dt$

$\displaystyle = 5\sqrt{x^2+6x+13} - 12\int \text{τεμ}\,t\,dt =$

$\displaystyle = 5\sqrt{x^2+6x+13} - 12\ln|\text{τεμ}\,t + \text{εφ}\,t| + C =$

$\displaystyle = 5\sqrt{x^2+6x+13} - 12\ln\left|\sqrt{\left(\frac{x+3}{2}\right)^2+1} + \frac{x+3}{2}\right| + C$

81) $\displaystyle\int \frac{3x-5}{\sqrt{x(4-x)}}\,dx = \int \frac{3x-5}{\sqrt{4x-x^2}} =$ $\quad d(4x-x^2)=(-2x+4)dx$

$\displaystyle = \int \frac{-\frac{3}{2}(-2x+4)+1}{\sqrt{4-(x-2)^2}}\,dx = -\frac{3}{2}\int \frac{d(4x-x^2)}{\sqrt{4x-x^2}} + \int \frac{dx}{\sqrt{4-(x-2)^2}} =$

$= -3\sqrt{4x-x^2} + \text{τοξημ}(x-2) + C$

82) $\displaystyle\int \frac{7x+11}{x^2+8x+12}\,dx =$ $\qquad d(x^2+8x+12)=(2x+8)\,dx$

$$= \int \frac{\frac{7}{2}(2x+8) - 17}{x^2+8x+12}\,dx = \frac{7}{2}\int \frac{(2x+8)\,dx}{x^2+8x+12} - 17\int \frac{dx}{x^2+8x+12} =$$

$$= \frac{7}{2}\int \frac{d(x^2+8x+12)}{x^2+8x+12} - 17\int \frac{dx}{(x+6)(x+2)} =$$

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$$= \frac{7}{2}\ln|x^2+8x+12| + \frac{17}{4}\int\frac{dx}{x+6} - \frac{17}{4}\int\frac{dx}{x+2}$$

$$\frac{1}{(x+6)\cdot(x+2)} = \frac{A}{x+6} + \frac{B}{x+2}$$

$$1 = A(x+2) + B(x+6)$$

$$= \frac{7}{2}\ln|x^2+8x+12| + \frac{17}{4}\ln|x+6| - \frac{17}{4}\ln|x+2| + c$$

για $x=-2 \Leftrightarrow 1 = 4B \Leftrightarrow B = \frac{1}{4}$

$$= \frac{7}{2}\ln|x^2+8x+12| + \frac{17}{4}\ln\left|\frac{x+6}{x+2}\right| + c$$

για $x=-6 \Leftrightarrow 1 = -4A \Leftrightarrow A = -\frac{1}{4}$

83) $\displaystyle\int \frac{3x}{\sqrt{12x-9x^2-2}}\,dx =$ $\qquad d(12x-9x^2-2) = (-18x+12)\,dx$

$$= \int \frac{-\frac{3}{18}(-18x+12)+2}{\sqrt{12x-9x^2-2}} = -\frac{3}{18}\int\frac{(-18x+12)\,dx}{\sqrt{12x-9x^2-2}} + 2\int\frac{dx}{\sqrt{-[9x^2-12x-2]}} =$$

$$= \frac{-3}{18}\int\frac{d(12x-9x^2-2)}{\sqrt{12x-9x^2-2}} + 2\int\frac{dx}{\sqrt{-[(3x-2)^2-2}} =$$

$$= \frac{-3}{18}\int\frac{d(12x-9x^2-2)}{\sqrt{12x-9x^2-2}} + 2\int\frac{dx}{\sqrt{2-(3x-2)^2}} =$$

$$= \frac{-3}{18}\sqrt{12x-9x^2-2} + \frac{2}{\sqrt{2}}\int\frac{dx}{\sqrt{1-\left(\frac{3x-2}{\sqrt{2}}\right)^2}} =$$

$$= \frac{-3}{18}\sqrt{12x-9x^2-2} + \frac{2}{\sqrt{2}\cdot 3}\,\text{τοξημ}\left(\frac{3x-2}{2}\right) + c$$

84) $\int \frac{3+x}{(1-x)(1+3x)}dx =$

$\frac{3+x}{(1-x)\cdot(1+x)} = \frac{A}{1-x} + \frac{B}{1+3x} \Leftrightarrow$

$3+x = A(1+3x)+B(1-x)$

για $x=1 \Leftrightarrow 4=4A \Leftrightarrow \boxed{A=1}$

για $x=0 \Leftrightarrow 3=A+B \Leftrightarrow 3=1+B \Leftrightarrow$ $B=2$

$= \int \frac{dx}{1-x} + 2\int \frac{dx}{1+3x} =$

$= -\ln|1-x| + \frac{2}{3}\ln|1+3x| + c$

85) $\int \frac{x^2+2x-1}{(x+1)(x^2+1)}dx =$

$\frac{x^2+2x-1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+\Gamma}{x^2+1}$

$x^2+2x-1 = A(x^2+1)+(Bx+\Gamma)(x+1)$

για $x=-1 \Leftrightarrow 1-2-1=2A \Leftrightarrow \boxed{A=-1}$

για $x=0 \Leftrightarrow -1=A+\Gamma \Leftrightarrow -1=-1+\Gamma \Rightarrow \boxed{\Gamma=0}$

συντ. x^2: $1=A+B \Leftrightarrow 1=-1+B \Leftrightarrow \boxed{B=2}$

$= \int \frac{dx}{x+1} + \int \frac{2x\,dx}{x^2+1} =$

$= -\int \frac{dx}{x+1} + \int \frac{d(x^2+1)}{x^2+1} =$

$= -\ln|x+1| + \ln|x^2+1| + c$

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86) $\int \frac{x^2+x-3}{(x-2)^2\cdot(x-1)}dx =$

$\frac{x^2+x-3}{(x-1)(x-2)^2} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{\Gamma}{(x-2)^2}$

$x^2+x-3 = A(x-2)^2+B(x-1)(x-2)+\Gamma(x-1)$

για $x=1 \Rightarrow -1=A \Leftrightarrow \boxed{A=-1}$

για $x=2 \Rightarrow 3=\Gamma \Rightarrow \boxed{\Gamma=3}$

συντ. x^2: $1=A+B \Leftrightarrow 1=-1+B \Leftrightarrow \boxed{B=2}$

$= -\int \frac{dx}{x-1} + 2\int \frac{dx}{x-2} + 3\int \frac{dx}{(x-2)^2}$

$= -\ln|x-1| + 2\ln|x-2| + 3\int u^{-2}du =$

θέτω $x-2=u \Leftrightarrow dx=du$

$= -\ln|x-1| + 2\ln|x-2| + \frac{3u^{-1}}{-1} + c = -\ln|x-1| + 2\ln|x-2| - \frac{3}{u-2} + c$

87) $\int \sqrt{9-x^2}\,dx =$

θέτω $x = 3\eta\mu t$

$dx = 3\sigma\upsilon\nu t\,dt$

$= \int \sqrt{9-9\eta\mu^2 t}\cdot 3\sigma\upsilon\nu t\,dt = 9\int \sigma\upsilon\nu^2 t\,dt = \frac{9}{2}\int (1+\sigma\upsilon\nu 2t)\,dt =$

$= \frac{9}{2}t + \frac{9}{4}\eta\mu 2t + c = \frac{9}{2}\tau o\xi\eta\mu\frac{x}{3} + \frac{9}{4}\cdot 2\eta\mu t\cdot\sigma\upsilon\nu t + c$

88) $\int \sqrt{16+x^2}\,dx =$ Θέτω $x = 4\,\text{εφ}t$

$dx = 4\,\text{τεμ}^2 t\,dt$

$= \int \sqrt{16+16\,\text{εφ}^2 t} \cdot 4\,\text{τεμ}^2 t\,dt =$

$= 16\int \text{τεμ}^3 t\,dt = 16\int \text{τεμ}^2 t \cdot \text{τεμ}\,t\,dt = 16\int (1+\text{εφ}^2 t)\,\text{τεμ}\,t\,dt =$

$= 16\int \text{τεμ}\,t\,dt + 16\int \text{εφ}^2 t\,\text{τεμ}\,t\,dt =$

$= 16\int \text{τεμ}\,t\,dt + 16\int \text{εφ}\,t \cdot \text{εφ}\,t\,\text{τεμ}\,t\,dt =$

$= 16\int \text{τεμ}\,t\,dt + 16\int \text{εφ}\,t\,d(\text{τεμ}\,t) =$

$= 16\int \text{τεμ}\,t\,dt + 16\,\text{εφ}\,t \cdot \text{τεμ}\,t - 16\int \text{τεμ}\,t\,d(\text{εφ}\,t) =$

$= 16\ln|\text{τεμ}\,t + \text{εφ}\,t| + 16\,\text{εφ}\,t \cdot \text{τεμ}\,t - 16\int \text{τεμ}^3 t\,dt \Leftrightarrow$

$32\int \text{τεμ}^3 dt = 16\ln|\text{τεμ}\,t + \text{εφ}\,t| + 16\,\text{εφ}\,t \cdot \text{τεμ}\,t$

$\int \text{τεμ}^3 t\,dt = \frac{16}{32}\ln|\text{τεμ}\,t + \text{εφ}\,t| + \frac{16}{32}\,\text{εφ}\,t \cdot \text{τεμ}\,t$

Άρα $\int \sqrt{16+x^2}\,dx = 16\int \text{τεμ}^3 t\,dt =$

$= 8\ln|\text{τεμ}\,t + \text{εφ}\,t| + 8\,\text{εφ}\,t \cdot \text{τεμ}\,t + c$

$= 8\ln\left|\sqrt{\frac{x^2}{16}+1} + \frac{x}{4}\right| + 8\cdot\frac{x}{4}\cdot\sqrt{\frac{x^2}{16}+1} + c$

89) $\int \sqrt{6x-x^2-8}\, dx = \int \sqrt{-(x^2-6x+8)}\, dx =$

$= \int \sqrt{-[(x-3)^2-1]}\, dx = \int \sqrt{1-(x-3)^2}\, dx =$

θέτω $x-3 = ημθ$

$dx = συνθ\, dθ$

$= \int \sqrt{1-ημ^2θ} \cdot συνθ\, dθ =$

$= \int συν^2θ\, dθ = \frac{1}{2}\int (1+συν2θ)\, dθ = \frac{θ}{2} + \frac{ημ2θ}{4} + c =$

$= \frac{τοξημ(x-3)}{2} + \frac{2ημθ \cdot συνθ}{4} + c =$

$= \frac{τοξημ(x-3)}{2} + \frac{(x-3) \cdot \sqrt{1-(x-3)^2}}{2} + c$

90) $\int \sqrt{9x^2-4}\, dx =$

θέτω $x = \frac{2}{3} τεμθ$

$dx = \frac{2}{3} τεμθ \cdot εφθ\, dθ$

$= \int \sqrt{4 τεμ^2θ - 4} \cdot \frac{2}{3} τεμθ\, εφθ\, dθ =$

$= \frac{4}{3}\int εφ^2θ\, τεμθ\, dθ = \frac{4}{3}\int εφθ \cdot εφθ \cdot τεμθ\, dθ =$

$= \frac{4}{3}\int εφθ\, d(τεμθ) = \frac{4}{3} τεμθ \cdot εφθ - \frac{4}{3}\int τεμ^3θ\, dθ$

$= \frac{4}{3} τεμθ \cdot εφθ - \frac{2}{3} \ln|τεμθ + εφθ| - \frac{2}{3} τεμθ \cdot εφθ + c$

$= \frac{2}{3} τεμθ \cdot εφθ - \frac{2}{3} \ln|τεμθ + εφθ| + c \overset{*}{=}$

$\boxed{\int τεμ^3θ\, dθ = \frac{1}{2} \ln|τεμθ + εφθ| + \frac{1}{2} τεμθ \cdot εφθ}$ από την 88 άσκηση.

$\overset{*}{=} \cancel{\frac{2}{3}} \cdot \cancel{\frac{3}{2}} x \cdot \sqrt{\frac{9}{4}x^2 - 1} - \frac{2}{3} \ln\left| \frac{3}{2}x + \sqrt{\frac{9}{4}x^2 - 1} \right| + c$

$= \frac{x}{2}\sqrt{9x^2-4} - \frac{2}{3} \ln\left| \frac{3}{2}x + \frac{9x^2-4}{2} \right| + c$

91) $\displaystyle\int \frac{x^2}{\sqrt{x^2+4}}\,dx =$ Θέτω $x = 2\,\text{εφ}\,t$, $dx = 2\,\text{τεμ}^2 t\,dt$

$$= \int \frac{4\,\text{εφ}^2 t \cdot \cancel{2}\,\text{τεμ}^{\cancel{2}} t\,dt}{\cancel{2}\cdot \cancel{\text{τεμ}\,t}} = 4\int \text{εφ}^2 t \cdot \text{τεμ}\,t\,dt =$$

$$= 4\int \text{εφ}\,t\cdot\text{εφ}\,t\,\text{τεμ}\,t\,dt = 4\int \text{εφ}\,t\,d(\text{τεμ}\,t) =$$

$$= 4\,\text{εφ}\,t\cdot\text{τεμ}\,t - 4\int \text{τεμ}^3 t\,dt =$$

ΣΑΒΒΑΣ ΧΡΥΣΑΝΘΟΥ
ΜΑΘΗΜΑΤΙΚΟΣ

$$= 4\,\text{εφ}\,t\cdot\text{τεμ}\,t - \frac{4}{2}\ln|\text{τεμ}\,t+\text{εφ}\,t| - \frac{4}{2}\,\text{τεμ}\,t\cdot\text{εφ}\,t + C$$

$$= 2\,\text{εφ}\,t\cdot\text{τεμ}\,t - 2\ln|\text{τεμ}\,t+\text{εφ}\,t| + C$$

$$= \cancel{2}\,\frac{x}{\cancel{2}}\cdot\sqrt{\frac{x^2}{4}+1} - 2\ln\left|\sqrt{\frac{x^2+4}{4}} - \frac{x}{2}\right| + C$$

$$= \frac{x}{2}\sqrt{x^2+4} - 2\ln\left|\sqrt{\frac{x^2+4}{4}} - \frac{x}{2}\right| + C$$

92) $\displaystyle\int \frac{dx}{\text{συν}2x - 3\,\text{ημ}^2 x} = \int \frac{dx}{\text{συν}2x - \frac{3}{2}(1-\text{συν}2x)} =$

$$= \int \frac{2\,dx}{2\,\text{συν}2x - 3 + 3\,\text{συν}2x} = 2\int \frac{dx}{5\,\text{συν}2x - 3} =$$

$t = \text{εφ}\,x$
$x = \text{τοξεφ}\,t$
$dx = \dfrac{dt}{1+t^2}$

$$= 2\int \frac{\frac{1}{1+t^2}\,dt}{5\cdot\frac{1-t^2}{1+t^2} - 3} = 2\int \frac{\frac{1}{1+t^2}\,dt}{\frac{5-5t^2-3-3t^2}{1+t^2}} = 2\int \frac{dt}{2-8t^2} =$$

$$= \int \frac{dt}{1-4t^2} = \frac{1}{2}\int \frac{dt}{1-2t} + \frac{1}{2}\int \frac{dt}{1+2t} =$$

$$\frac{1}{(1-2t)\cdot(1+2t)} = \frac{A}{(1-2t)} + \frac{B}{1+2t}$$

$$= -\frac{1}{4}\ln|1-2t| + \frac{1}{4}\ln|1+2t| + C$$

$1 = A(1+2t) + B(1-2t)$

για $t = \frac{1}{2} \Leftrightarrow 1 = 2A \Leftrightarrow A = \frac{1}{2}$

για $t = -\frac{1}{2} \Leftrightarrow 1 = +2B \Leftrightarrow B = \frac{1}{2}$

$$= \frac{1}{4}\ln\left|\frac{1+2t}{1-2t}\right| + C = \frac{1}{4}\ln\left|\frac{1+2\,\text{εφ}\,x}{1-2\,\text{εφ}\,x}\right| + C$$

93) $\displaystyle\int \frac{dx}{1+\eta\mu x+\sigma\upsilon\nu x} =$

Θέτω $\varepsilon\varphi \frac{x}{2} = t$

$\frac{x}{2} = \tau o\xi\varepsilon\varphi\, t$

$dx = \frac{2dt}{1+t^2}$

$\eta\mu x = \frac{2t}{1+t^2}$, $\sigma\upsilon\nu x = \frac{1-t^2}{1+t^2}$ $\Big\}\ t = \varepsilon\varphi \frac{x}{2}$

$$= \int \frac{\frac{2dt}{1+t^2}}{1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}} = \int \frac{2dt}{1+\cancel{t^2}+2t+1-\cancel{t^2}} =$$

$$= \int \frac{2dt}{2+2t} = \int \frac{dt}{1+t} =$$

$$= \ln|1+t| + c = \ln\left|1+\varepsilon\varphi\frac{x}{2}\right| + c$$

94) $\displaystyle\int \frac{3dx}{5+4\sigma\upsilon\nu x+3\eta\mu x} = \int \frac{\frac{6dt}{1+t^2}}{5+4\cdot\frac{1-t^2}{1+t^2}+\frac{6t}{1+t^2}} =$ Θέτω $\varepsilon\varphi\frac{x}{2} = t$

$$= \int \frac{6dt}{5+5t^2+4-4t^2+6t} = \int \frac{6dt}{t^2+6t+9} = 6\int \frac{dt}{(t+3)^2} =$$

$$= 6\int (t+3)^{-2} d(t+3) = 6\frac{(t+3)^{-1}}{-1} + c = -\frac{6}{t+3} + c$$

$$= -\frac{6}{\varepsilon\varphi\frac{x}{2}+3} + c$$

95) $\displaystyle\int \tau\varepsilon\mu x\, dx = \int \frac{dx}{\sigma\upsilon\nu x} = \int \frac{\frac{2dt}{\cancel{1+t^2}}}{\frac{1-t^2}{\cancel{1+t^2}}} = 2\int \frac{dt}{1-t^2} =$

$$2\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c = \ln\left|\frac{\varepsilon\varphi\frac{x}{2}-1}{\varepsilon\varphi\frac{x}{2}+1}\right| + c$$

96) $\displaystyle\int \sigma\varphi x\, dx = \int \frac{dx}{\eta\mu x} = \int \frac{\frac{2dt}{\cancel{1+t^2}}}{\frac{2t}{\cancel{1+t^2}}} = \int \frac{dt}{t} = \ln|t| + c$

$\ln|\varepsilon\varphi x| + c$

97) $\int \frac{dx}{x\sqrt{x^2+1}} = \int \frac{\cancel{u}\cdot du}{x\cdot x\cdot \cancel{u}} =$

Θέτω $\sqrt{x^2+1}=u$

$x^2+1=u^2$

$\cancel{2}x\,dx = \cancel{2}u\,du$

$= \int \frac{du}{x^2} = \int \frac{du}{u^2-1} =$

$= \frac{1}{2} \ln\left|\frac{u-1}{u+1}\right| + c = \frac{1}{2} \ln\left|\frac{\sqrt{x^2+1}-1}{\sqrt{x^2-1}+1}\right| + c$

98) $\int \frac{dx}{x^2\sqrt{1+x^2}} =$

Θέτω $x = εφt$

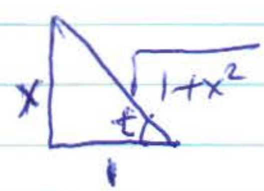
$dx = τεμ^2t\,dt$

$= \int \frac{τεμ^{\cancel{2}}t\,dt}{εφ^2t\cdot \cancel{τεμt}} = \int \frac{συν^{\cancel{2}}t}{\cancel{συνt}\,ημ^2t}\,dt =$

$= \int \frac{συνt}{ημ^2t}\,dt = \int συνt\cdot ημ^{-2}t\,dt = \int ημ^{-2}t\,d(ημt) =$

$= \frac{ημ^{-1}t}{-1} + c = -\frac{1}{ημt} + c =$

$= -\frac{1}{\frac{x}{\sqrt{1+x^2}}} + c = -\frac{\sqrt{1+x^2}}{x} + c$



99) $\int\sqrt{\frac{x-2}{x+2}}\,dx = \int \frac{\sqrt{x-2}}{\sqrt{x+2}}\cdot\frac{\sqrt{x-2}}{\sqrt{x-2}}\,dx = \int\frac{x-2}{\sqrt{x^2-4}}\,dx$

$= \int \frac{\frac{1}{2}(2x)-2}{\sqrt{x^2-4}}\,dx =$

$d(x^2-4) = 2x\,dx$

$= \frac{1}{2}\int\frac{2x\,dx}{\sqrt{x^2-4}} - 2\int\frac{dx}{\sqrt{x^2-4}} =$

$= \frac{1}{2}\sqrt{x^2-4} - 2\int\frac{dx}{\sqrt{x^2-4}} =$

Θέτω $x = 2τεμθ$

$dx = 2τεμθ\,εφθ\,dθ$

$= \frac{1}{2}\sqrt{x^2-4} - 2\int\frac{\cancel{2}τεμθ\,\cancel{εφθ}\,dθ}{\cancel{2}\,\cancel{εφθ}} = \frac{1}{2}\sqrt{x^2-4} - 2\ln|τεμθ+εφθ| + c =$

$= \frac{1}{2}\sqrt{x^2-4} - 2\ln\left|\frac{x}{2}+\sqrt{\frac{x^2}{4}-1}\right| + c$

100) $\int \frac{ημ^3x}{συν^2x}\,dx = \int \frac{1-συν^2x}{συν^2x}\,d(συνx) =$

$= \int \frac{d(συνx)}{συν^2x} - \int d(συνx) = -\frac{1}{συνx} - συνx + c$

101) $\int \frac{ημ^4x}{συν^6x}\,dx = \int \frac{ημ^4x}{συν^4x\, συν^2x}\,dx = \int εφ^4x\,(εφx)'\,dx = \frac{εφ^5x}{5} + c$

102) $\int \frac{dx}{1+\sqrt{x-2}} = \int \frac{2u\,du}{1+u} =$ θέτω $\sqrt{x-2} = u$

$x-2 = u^2$

$dx = 2u\,du$

$= \int 2du - 2\int \frac{du}{u+1} =$

$\begin{array}{r|l} 2u & u+1 \\ \hline -2u-2 & 2 \\ -2 & \end{array}$

$= 2u - 2\ln|u+1| + c =$

$= 2\sqrt{x-2} - 2\ln|\sqrt{x-2}+1| + c$

103) $\int \frac{e^{-\frac{2}{x}}}{x^3}\,dx = \int \frac{e^{-\frac{2}{x}}}{x^2 \cdot x}\,dx =$ θέτω $\frac{2}{x} = u$

$-\frac{2}{x^2}dx = du$

$= -\frac{1}{2}\int e^{-u}\cdot \frac{u}{2}\cdot du = -\frac{1}{4}\int e^{-u}\cdot u\,du =$

$= +\frac{1}{4}\int u\,d(e^{-u}) = \frac{1}{4}u\cdot e^{-u} + \frac{1}{4}\int e^{-u}\,du =$

$= \frac{1}{4}\cdot u\cdot e^{-u} + \frac{1}{4}e^{-u} - c = \frac{1}{4}\cdot\frac{2}{x}e^{-\frac{2}{x}} - \frac{1}{4}e^{-\frac{2}{x}} + c$

104) $\int (\varepsilon\varphi^5 x + \varepsilon\varphi^7 x)\,dx =$

$= \int \varepsilon\varphi^5 x\,dx + \int \varepsilon\varphi^7 x\,dx$

Γενικός τύπος (Άσκηση 51)

$$\boxed{I_\nu = \int \varepsilon\varphi^\nu x\,dx = \frac{\varepsilon\varphi^{\nu-1} x}{\nu-1} - I_{\nu-2}}$$

$$I_7 = \int \varepsilon\varphi^7 x\,dx = \frac{\varepsilon\varphi^6 x}{6} - \int \varepsilon\varphi^5 x\,dx$$

$$I_5 = \int \varepsilon\varphi^5 x\,dx = \frac{\varepsilon\varphi^4 x}{4} - \int \varepsilon\varphi^3 x\,dx$$

Άρα $\int (\varepsilon\varphi^5 x + \varepsilon\varphi^7 x)\,dx = \int \varepsilon\varphi^7 x\,dx + \int \varepsilon\varphi^5 x\,dx =$

$$= \frac{\varepsilon\varphi^6 x}{6} - \cancel{\int \varepsilon\varphi^5 x\,dx} + \cancel{\int \varepsilon\varphi^5 x\,dx} = \frac{\varepsilon\varphi^6 x}{6} + c$$

105) $\int \frac{dx}{1+\sigma\varphi x} = \int \frac{dt}{\quad} =$ θέτω $\sigma\varphi x = t$

$-\sigma\tau\varepsilon\mu^2 x\,dx = dt$

$1 + \sigma\varphi^2 x = \sigma\tau\varepsilon\mu^2 x$

$1 + t^2 = \sigma\tau\varepsilon\mu^2 x$

$$= -\int \frac{dt}{(1+t)\cdot(1+t^2)} =$$

$$= -\frac{1}{2}\int \frac{dt}{1+t} - \int \frac{-\frac{1}{2}t + \frac{1}{2}}{1+t^2}\,dt = \qquad \frac{1}{(1+t)(1+t^2)} = \frac{A}{1+t} + \frac{Bt+\Gamma}{1+t^2}$$

$1 = A(1+t^2) + (Bt+\Gamma)(1+t)$

$$= -\frac{1}{2}\int \frac{dt}{1+t} - \frac{1}{2}\int \frac{-t+1}{1+t^2}\,dt =$$ για $t=-1 \Leftrightarrow 1 = 2A \Leftrightarrow A = \frac{1}{2}$

για $t=0 \Leftrightarrow 1 = A + \Gamma \Leftrightarrow \Gamma = \frac{1}{2}$

$$= -\frac{1}{2}\int \frac{dt}{1+t} + \frac{1}{2}\int \frac{t\,dt}{1+t^2} - \frac{1}{2}\int \frac{dt}{1+t^2} =$$ συντ. t^2 $0 = A + B \Leftrightarrow \boxed{B = -\frac{1}{2}}$

$$= -\frac{1}{2}\int \frac{dt}{1+t} + \frac{1}{4}\int \frac{d(1+t^2)}{1+t^2} - \frac{1}{2}\int \frac{dt}{1+t^2} =$$ $d(1+t^2) = 2t\,dt$

$$= -\frac{1}{2}\ln|1+t| + \frac{1}{4}\ln(1+t^2) - \frac{1}{2}\,\tau o\xi\varepsilon\varphi\, t + c$$

106) $\int \text{Τεμ}^3\theta\, d\theta$ έχει λύθει άσκηση 88

107) $\int e^{3-2\ln x}\, dx = \int \frac{e^3}{e^{2\ln x}}\, dx = e^3 \int \frac{dx}{e^{\ln x^2}} = e^3 \int \frac{dx}{x^2} =$

$= -\frac{e^3}{x} + c$

108) $\int \ln(x+\sqrt{x^2+1})\, dx = x\cdot(x+\sqrt{x^2+1}) - \int x\, d\ln(x+\sqrt{x^2+1})$

$= x\cdot(x+\sqrt{x^2+1} - \int x \frac{\left(1+\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}\right)}{x+\sqrt{x^2+1}}\, dx =$

$= x(x+\sqrt{x^2+1} - \int x \frac{\left(\frac{\cancel{\sqrt{x^2+1}+x}}{\sqrt{x^2+1}}\right)}{\cancel{x+\sqrt{x^2+1}}}\, dx =$

1

$= x(x+\sqrt{x^2+1} - \int x \frac{dx}{\sqrt{x^2+1}} =$

$\sqrt{x^2+1} = u$
$x^2+1 = u^2$
$\cancel{2}x\,dx = \cancel{2}u\,du$

$= x(x+\sqrt{x^2+1} - \int \frac{\cancel{u}\,du}{\cancel{u}} =$

$= x(x+\sqrt{x^2+1} - u + c = x(x+\sqrt{x^2+1} - \sqrt{x^2+1} + c$

104) $\int \sqrt{e^x+1}\, dx =$

θέτω $\sqrt{e^x+1} = u$
$e^x+1 = u^2$
$e^x = u^2-1$
$e^x dx = 2u\,du$

$= \int \frac{u\cdot 2u\,du}{u^2-1} = 2\int \frac{u^2\,du}{u^2-1} =$

$= 2\int \frac{(u^2-1+1)\,du}{u^2-1} = 2\int \frac{u^2-1}{u^2-1}\,du + 2\int \frac{du}{u^2-1} =$

$2u + \frac{\cancel{2}}{\cancel{2}}\int \frac{du}{u-1} - \frac{2}{2}\int \frac{du}{u+1} =$

$\frac{1}{u^2-1} = \frac{A}{u-1} + \frac{B}{u+1}$

$= 2u + \ln\left|\frac{u-1}{\cdots}\right| + c = 2\sqrt{x^2+1} + \ln\left|\sqrt{e^x+1} - 1 \cdots\right| + c$

$1 = A(u+1) + B(u-1)$

110) $\int \frac{dx}{\sqrt{x-x^2}} = \int \frac{dx}{\sqrt{-(x^2-x+\frac{1}{4}-\frac{1}{4})}} = \int \frac{dx}{\sqrt{-(x-\frac{1}{2})^2+\frac{1}{4}}} =$

$= \int \frac{dx}{\sqrt{\frac{1}{4}-(x-\frac{1}{2})^2}} = \text{τοξημ}(2x-1) + c$

111) $\int \frac{x^3}{x^2-4}\,dx = \int \left(x + \frac{4x}{x^2-4}\right)dx =$

$$\begin{array}{r|l} x^3 & x^2-4 \\ \hline \cancel{-x^3}+4x & x \\ \hline 4x & \end{array}$$

$= \int x\,dx + \int \frac{4x}{x^2-4}\,dx =$

$d(x^2-4) = 2x\,dx$

$= \int x\,dx + \frac{4}{2}\int \frac{d(x^2-4)}{x^2-4} = \frac{x^2}{2} + \frac{1}{2}\ln|x^2-4| + c$